Assignment 8 Solutions

electric field flux for moving point charge:

$S(r)$ = surface of sphere, radius r

$$\Phi = \oint_{S(r)} \vec{E}\cdot d\vec{a} = \int r^2 \underbrace{d\phi}_{2\pi} \sin\theta \, d\theta \, \frac{Kq}{r^2} \frac{1}{\gamma^2} \frac{1}{(1-\beta^2\sin^2\theta)^{3/2}}$$

$$= 2\pi K q \int_0^{\pi} \frac{\sin\theta \, d\theta}{\gamma^2 (1-\beta^2 \sin^2\theta)^{3/2}}$$

$$2\pi K = \frac{1}{2\epsilon_0}, \quad \sin\theta \, d\theta = -d(\underbrace{\cos\theta}_{u})$$

$$1-\beta^2\sin^2\theta = 1-\beta^2+\beta^2 u^2$$

$$\frac{1}{\gamma^2} = 1-\beta^2$$

$$\Phi = \frac{q}{2\epsilon_0} \int_{-1}^{1} \frac{(1-\beta^2)(+du)}{\left((1-\beta^2)+\beta^2 u^2\right)^{3/2}}$$

useful "derivative fact":

$$\frac{d}{du}\left(\frac{u}{\sqrt{A+Bu^2}}\right) = \frac{1}{\sqrt{A+Bu^2}} - \frac{1}{2}\frac{2Bu^2}{(A+Bu^2)^{3/2}}$$

$$= \frac{A+Bu^2 - Bu^2}{(A+Bu^2)^{3/2}}$$

$$= \frac{A}{(A+Bu^2)^{3/2}}$$

set $A = 1-\beta^2$, $B = \beta^2$

$$\Phi = \frac{q}{2\epsilon_0}\int_{-1}^{1}\frac{A\,du}{(A+Bu^2)^{3/2}} = \frac{q}{2\epsilon_0}\left.\frac{u}{\sqrt{A+Bu^2}}\right|_{-1}^{1}$$

$$= \frac{q}{2\epsilon_0}\left(\frac{1}{\sqrt{A+B}} - \frac{-1}{\sqrt{A+B}}\right) = \frac{q}{2\epsilon_0}\times 2$$

$$= q/\epsilon_0 \checkmark$$

field-line pattern of oscillating point charge:

At $t=0$ the charge is at $x=0$ and moving along the x-axis with velocity $+c/2$.
Define the following times & positions:

t_a : times in the past when the charge accelerated (reversed its velocity)

x_a : positions of charge at the times t_a

x_+ : position of light pulse moving with velocity $+c$ and launched at x_a & t_a, now at $t=0$.

x_- : same as x_+, but for light pulse moving with velocity $-c$.

$$X_+ = X_a + c(t - t_a), \quad X_- = X_a - c(t - t_a)$$

X_0 : extrapolated position of charge if it didn't undergo accelerations after t_a

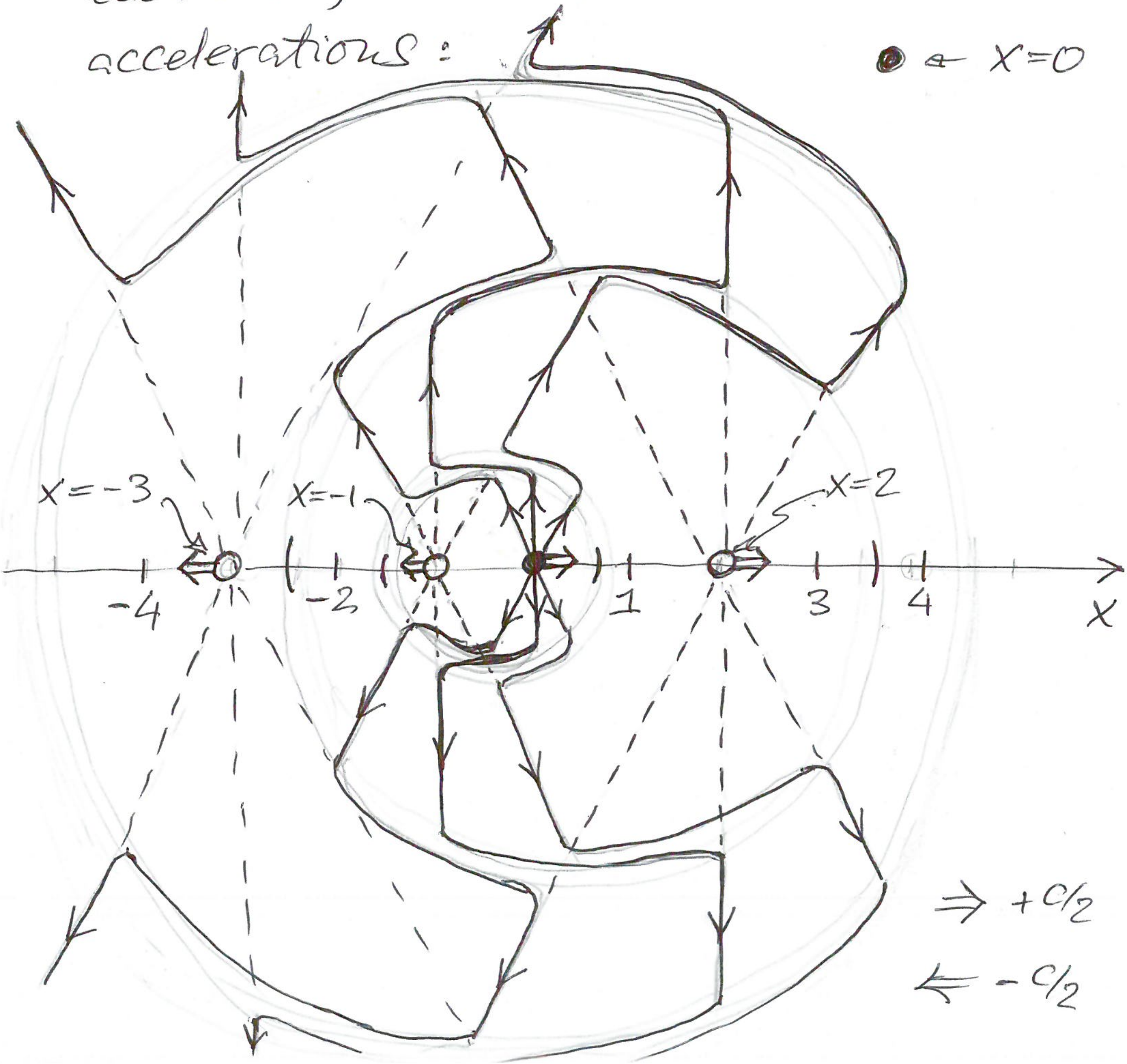
$X_0 = X_a + \frac{c}{2}(t - t_a)$ if moving with pos. velocity

$X_0 = X_0 - \frac{c}{2}(t - t_a)$ if neg. velocity

Here is a table of these times/positions for $t=0$. Times are in units of ns, positions in ft., so $c \cong 1$.

t_a	-1	-3	-5	-7
X_a	$-1/2$	$+1/2$	$-1/2$	$+1/2$
X_+	$1/2$	$7/2$	$9/2$	$15/2$
X_-	$-3/2$	$-5/2$	$-11/2$	$-13/2$
X_0	0	-1	$+2$	-3

The positions x_+ & x_- are the intersection of each "light-pulse shell" with the x-axis and x_0 is where the field lines would converge inside each shell, in the absence of later accelerations:



Charge in uniform electric field

$$\frac{d\vec{P}}{dt} = qE\,\hat{x} \quad , \quad \vec{P}(0) = 0 \quad \text{(at rest when } t=0\text{)}$$

$$P_y = P_z = 0$$

$$P_x = qEt$$

$$P_x = \frac{mv_x}{(1 - v_x^2/c^2)^{1/2}}$$ solve for v_x in terms of P_x

$$v_x = \frac{P_x/m}{(1 + (P_x/mc)^2)^{1/2}}$$

$$= \frac{\left(\frac{qE}{m}\right)t}{\left(1 + \left(\frac{qE}{mc}\right)^2 t^2\right)^{1/2}}$$

define a "time" $\tau = \frac{mc}{qE}$

$$v_x = \frac{c(t/\tau)}{(1 + (t/\tau)^2)^{1/2}}$$

From the last expression we see that for $t \to +\infty$, $v_x \to c$, and v_x is already close to c when $t \gg \tau$. In the infinite past, or $t \to -\infty$, $v_x \to -c$. In other words, the charge had been moving with negative, near-light velocity before the electric field brought it to rest, instantaneously, at $x=0$ and time $t=0$.

$$x = \int_0^t v_x \, dt = c \int_0^t \frac{(t/\tau)\,dt}{\left(1+(t/\tau)^2\right)^{1/2}}$$

$$= c\tau \sqrt{1+t^2/\tau^2}\,\Big|_0^t = c\tau\left(\sqrt{1+(t/\tau)^2} - 1\right)$$