Solutions for problem 1

Below is the list of equations where you were asked to circle those that apply **only in special situations or particular models**. When doing these a good strategy is to ask yourself if we have seen a different version of the equation, in lecture or in a homework assignment, that applied to a different situation. If that is the case, then the equation is not general or fundamental and should be circled.

$$\boxed{\varphi = K\,\frac{q}{r}}$$

This looks like a formula for electrical potential, but of course it only applies to a point charge. We have seen other examples of φ, such as the linear function that applies in a uniform field:

$$\boxed{V = Ed}$$

Here d is the distance between two points along a field line in a uniform field and V is the potential difference between those points.

$$\boxed{\nabla \cdot \mathbf{E} = 0}$$

This should be circled because it is a special case of the local Gauss law when the charge density ρ is zero.

$$C = \frac{Q}{V}$$

Recall that the capacitance C expresses the general property that the potential on a conductor (relative to infinity or another conductor) is proportional to the charge on the conductor. It applies for all shapes and sizes of capacitor (we covered two examples).

$$-\nabla^2\varphi = \rho/\epsilon_0$$

This is the most general form of the local Gauss law, expressed in terms of the potential instead of the electric field, which are always related by:

$$\mathbf{E} = -\nabla\varphi$$

$$\boxed{R = \rho\,\frac{L}{A}}$$

This formula for resistance applies only in the uniform field case, such as the space between the electrodes of a battery when these are parallel plates. We have seen another geometry in homework 6 and yet another geometry comes up in problem 4 of the prelim.

$$\int_{\mathcal{V}} \nabla \cdot \mathbf{v}\; d^3\mathbf{r} = \oint_{\mathrm{boundary}(\mathcal{V})} \mathbf{v} \cdot d\mathbf{a}$$

This is the statement of the divergence theorem, a general mathematical fact. One student (with a sense of humor?) circled this with the explanation that it only applies in three dimensions (and received credit).

$$\boxed{C = \epsilon_0\,\frac{A}{d}}$$

This formula for capacitance only applies in the parallel plate geometry. There is a different formula, covered in lecture, when the conductors are spherical.

$$\boxed{\nabla \cdot \mathbf{j} = 0}$$

The general statement of local charge conservation is $\nabla \cdot \mathbf{j} = -\partial\rho/\partial t$, where ρ is the charge density. The equation above applies only in time-independent situations.

$$u = \frac{\epsilon_0}{2}|\mathbf{E}|^2$$

This is the general law for electric energy density. It makes no reference to the source of the electric field and in fact applies even in electromagnetic waves, a phenomenon covered at the end of the course that has no field sources at all.

$$R = V/I$$

Like the equation $C = Q/V$, this expresses the proportionality between the relevant quantities of another device, the voltage and current in a resistor.

$$\boxed{U = K \sum_{i<j} \frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|}}$$

This is the electric energy for a system of point charges, a special model for the charge density. You could not use this formula to calculate the electric energy of a hydrogen atom (homework 4).

$$\varphi(b) - \varphi(a) = -\int_{\text{path } a \to b} \mathbf{E} \cdot d\ell$$

This is another general statement of the relationship between potential and electric field, $\mathbf{E} = -\nabla\varphi$ being the other one.

$$\boxed{\mathbf{j} = nq\,\mathbf{v}_D}$$

We have seen examples of current density where the free charges are not all of the same type. In the electrolyte of the lead-acid battery the current arises from both drifting H^+ and SO_4^{-2} (with drift velocities differing in sign). The general formula would involve a sum over all the types of free charge.