Lecture 9

With the definition of the "Volt", or "energy per Coulomb of charge", we now have two common ways of expressing the units of electric field. Recall:

$$\vec{F} = q\vec{E} \quad (1)$$

$$-\vec{\nabla}\varphi = \vec{E} \quad (2)$$

From (1) the units of $\vec{E}$ are N/C while (2) tells us the units of $\vec{E}$ are V/m (Volt is the unit of φ). These unit combinations are, of course, equal; sometimes one is more convenient than the other.

Another unit, the "electron-volt" or eV, should not be confused with the Volt. Electron-volt is actually a unit of <u>energy</u>. It corresponds to the energy <u>change</u> when the unit of elementary charge $e \simeq 1.6 \times 10^{-19} C$ is moved between points that differe in electric potential by 1 Volt:

$$1 eV \simeq (1.6 \times 10^{-19} C) \underbrace{(1 J/C)}_{\text{Volt}}$$

$$\simeq 1.6 \times 10^{-19} J$$

The <u>LHC</u> is designed to accelerate ("Large Hadron Collider") protons up to energy 7 TeV (tera electron volts) which is equivalent to

$$7 \times \underset{\substack{\uparrow \\ \text{"tera"}}}{10^{12}} \times 1.6 \times 10^{-19} \simeq 10^{-6}\ J$$

A micro-Joule is a lot of energy for just one proton!

The electric potential function $\varphi(\vec{r})$ is very useful in understanding the static properties of <u>conductors</u>!

Conductors are materials that have <u>free</u> <u>charges</u>, and can take the form of solids, liquids, and gases. <u>Metals</u> are an important class of solid conductor, where the free charges are the conduction electrons. In electrolytes, or liquid conductors, the free charges are mobile ions. Gaseous conductors, where a significant fraction of atoms/molecules in a gas are ionized, are called plasmas. These are found naturally in the atmospheres of the sun and the planets ("ionosphere").

Electrostatics concerns itself with equilibrium configurations of free charge. Free charge in the presence of an electric field will not be in equilibrium;

it will accelerate as a result of the electric force. This simple fact leads to several strong conclusions about $\vec{E}$ and φ when we have a conductor in a static situation:

(1) $\vec{E} = 0$ (everywhere inside a conductor)

(2) $\varphi(\vec{r}) = \varphi_0$ (at all points $\vec{r}$ inside a conductor the potential has the same value)

(3) $\rho(\vec{r}) = 0$ (net free charge ρ is zero at all points $\vec{r}$ inside a conductor)

(4) $\vec{E} \perp$ surface

(if the conductor has a surface, the electric field just outside the surface is perpendicular to the surface)

Properties (1) and (2) are direct consequences of equilibrium and $\nabla\varphi = -\vec{E}$. Property (3) needs some elaboration. First, we can have a non-zero density of free charge and still have $\rho(\vec{r}) = 0$. The charge density $\rho(\vec{r})$ in Gauss's law refers to the <u>net</u> charge density, not the density of free charges. A conductor in equilibrium will have charges of both signs, at least one of which is free to move. The conduction electrons in a metal are free negative charges; they are matched by an exactly compensating density of positive charge fixed to the metal ions. When the two signs of charge do not exactly cancel and $\rho(\vec{r}) \neq 0$ inside the conductor, Gauss's law $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$ tells us there must be an $\vec{E}$ with a certain divergence which is possible only if $\vec{E}$ is nonzero. But $\vec{E}$ is everywhere zero inside a conductor, and so $0 = \vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$.

Property (4) concerns the equilibrium of free charges on the surface of a conductor. If $\vec{E}$ was not perpendicular, and had a component parallel to the surface, then the free charges at the surface would not be in equilibrium. The property of the potential that corresponds to this is

(5) $\varphi =$ const. contours just outside a conductor are parallel to the surface of the conductor

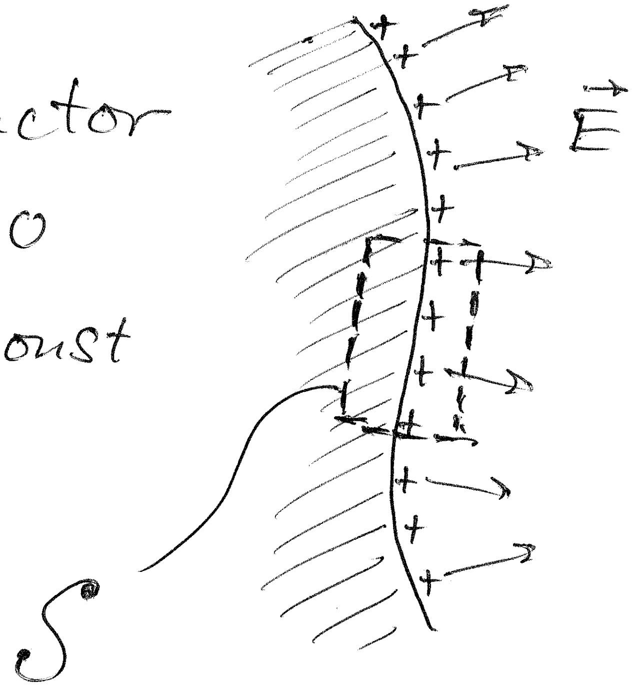
Recall that the gradient $\vec{\nabla}\varphi$ is always perpendicular to the contours of φ.

Interesting things can happen when a conductor has surfaces. We know that if the conductor has a non-zero net charge then none of it can be inside (by property (3)). Hence, in those situations all the net charge must be on the surface! At a surface the electric field can have a divergence while still being zero on the conductor-side of the surface. Here is a drawing that applies to the behavior at any conductor surface:

conductor

$\vec{E} = 0$

$\varphi = \text{const}$



The (positive) surface charge is the source of $\vec{E}$ that is everywhere perpendicular to the conductor surface. Shown in the drawing is a small ~~sa~~ closed surface S that cuts through some of the conductor's surface charge. We can make the part of S outside the conductor be either parallel or perpendicular to the conductor surface. There is zero flux of $\vec{E}$ through the perpendicular parts. The flux through the parallel part is just $|\vec{E}|A$, where A is the area of that part of $\mathcal{S}$. Keeping $\mathcal{S}$ small, the surface charge density σ will be nearly uniform inside $\mathcal{S}$, so the enclosed charge is $\sigma \cdot A$. The flux-form of Gauss's law tells us

$$\Phi = |\vec{E}|A = \frac{\sigma A}{\epsilon_0}$$

$$\Rightarrow |\vec{E}_{surf}| = \frac{\sigma}{\epsilon_0}$$