Lecture 6

Electric charge is known to have three distinct properties:

(1) conservation

(2) quantization

(3) invariance

Property (1) is simply that the algebraic sum of charges in a system is a conserved quantity. Later in the course we'll express this property in differential form (locally) and see how conservation is <u>necessary</u> in order for the Maxwell equations of electricity and magnetism to be consistent.

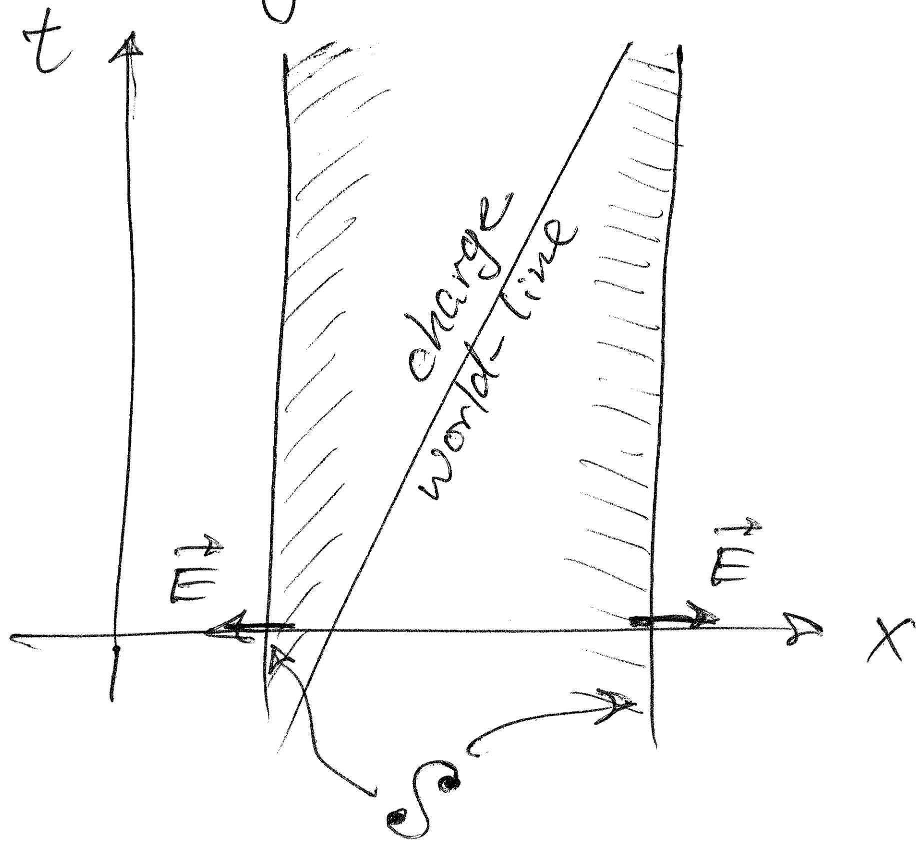
Property (2) is primarily an "observation", one that has tremendous implications for the structure of matter (periodic table of elements, chemistry, ...). It's hard to imagine a world where charge doesn't come in integer parcels of e. On the other hand, the equations of E & M would still be consistent without this property.

Property (3) has a connection with Gauss's law. The word "invariance" refers to "invariance with respect to transformations of space-time. As charge is a scalar, it is clearly unchanged by rotations. More interestingly, charge is also a <u>Lorentz-invariant</u>. With the help of Gauss's law we can define "charge" even when the source of charge (e.g. a particle) is moving:

$$\oint_S^{*} \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

* in any ~~frame~~ inertial frame of reference where the closed surface S is at rest.

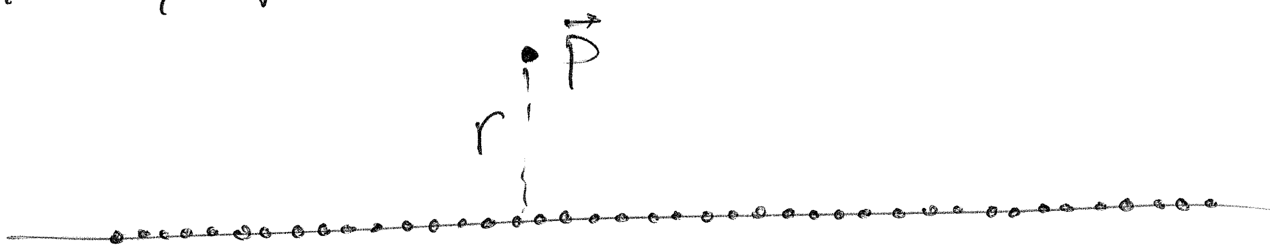
We had to add a foot- (4)
note (*) to Gauss's law in
a space-time setting because
an integral over $\mathcal{S}$ refers to
a particular choice of simul-
taneous events ("space") which
is frame dependent. Suppose
we had this situation (space-
time diagram):



The surface $\mathcal{S}$ is at rest in this frame ~~and~~ (only two points of it are shown) and the charge is moving. Gauss's law equates the flux of $\vec{E}$ in the rest frame of $\mathcal{S}$ to just the charge enclosed — it does not refer to the state of motion of the charge. In other words, the "charge" of say, an electron, is independent of its state of motion.

The Lorentz-invariance of charge is an experimental fact that has been tested to high precision. Atoms and nuclei comprise systems of charged particles (quantized in units of e) with high relative velocities ($v \sim .01c$ in atoms, $v \sim c$ in nuclei) and yet the electric field from these composite sources are always from that of a net charge that is precisely a quantized value. Motion appears to have no effect on the source strength of charged particles [Purcell gives experimental references.]

Gauss's law and symmetry considerations are sufficient to determine in a few simple steps the electric fields of highly symmetric distributions of charges. As an example, consider an "infinite" line of equal, and equally spaced, charges:



Consider the electric field $\vec{E}$ at some point $\vec{P}$ a distance r from the line, where r is much larger than the spacing of the charges. In this limit the charges might as well be modeled as a continuum (spacing $\to 0$) characterized simply by the amount of charge λ per unit length. For such a continuum source, the electric field should have the following symmetries:

(1) unchanged by rotations about the line

(2) unchanged by translations of the line

(3) unchanged by reflecting through a plane $\perp$ to line

(4) unchanged by reflecting through a plane containing the line

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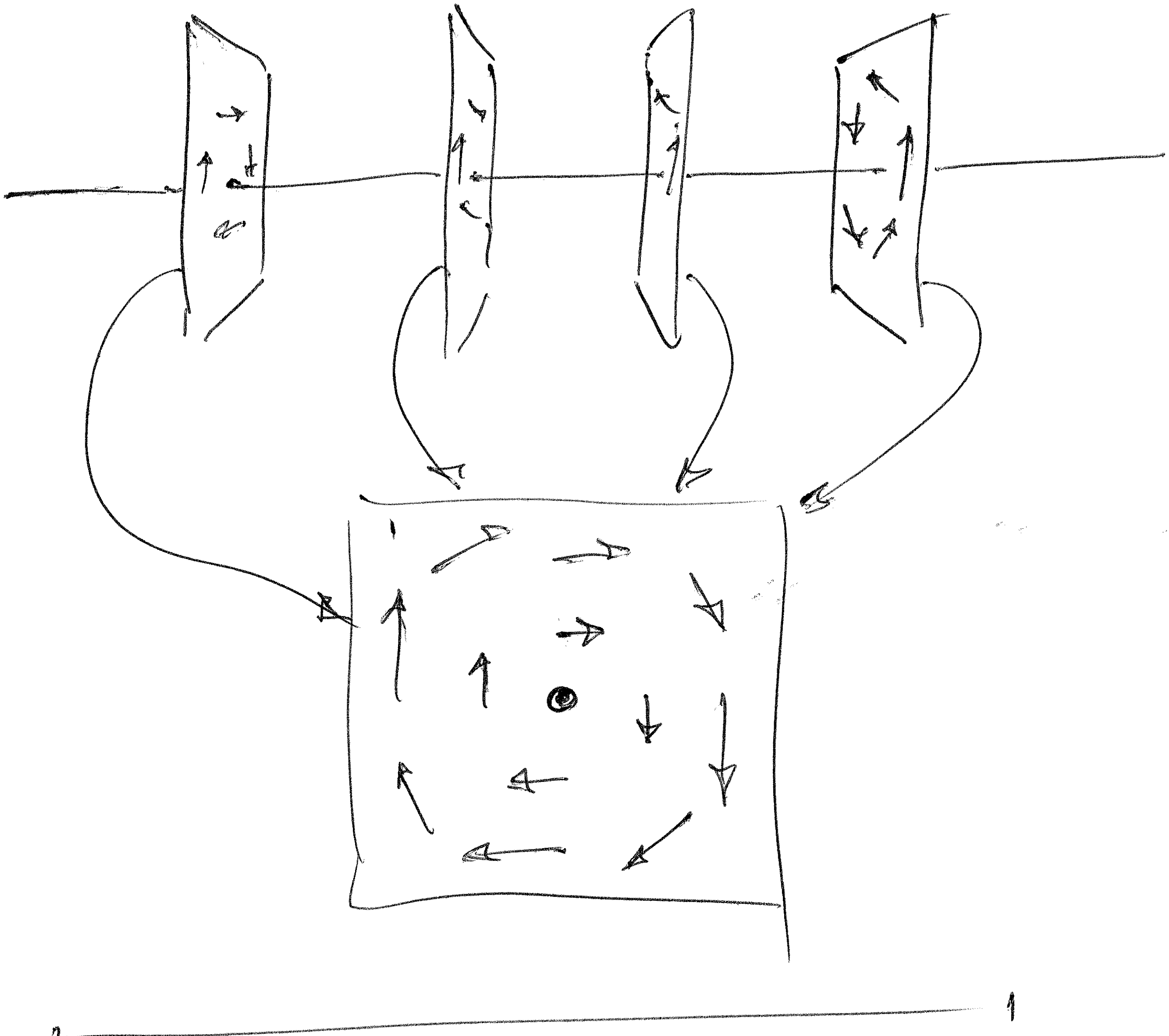
Q: Consider the following structure of $\vec{E}$; is it compatible with all four symmetries?



There is a unique configuration of $\vec{E}$ compatible with all the symmetries possessed by the charge ((1)–(4)) where $\vec{E}$ is purely radial (proportional to $\hat{r}$) and depends in magnitude only on r.