Lecture 4

The dipole electric field has the following structure:

$$\vec{D} = \left(\begin{array}{c}\text{scalar, inverse-}\\ \text{cube magnitude}\end{array}\right)\left(\begin{array}{c}\text{angularly-}\\ \text{dependent}\\ \text{vector}\end{array}\right)$$

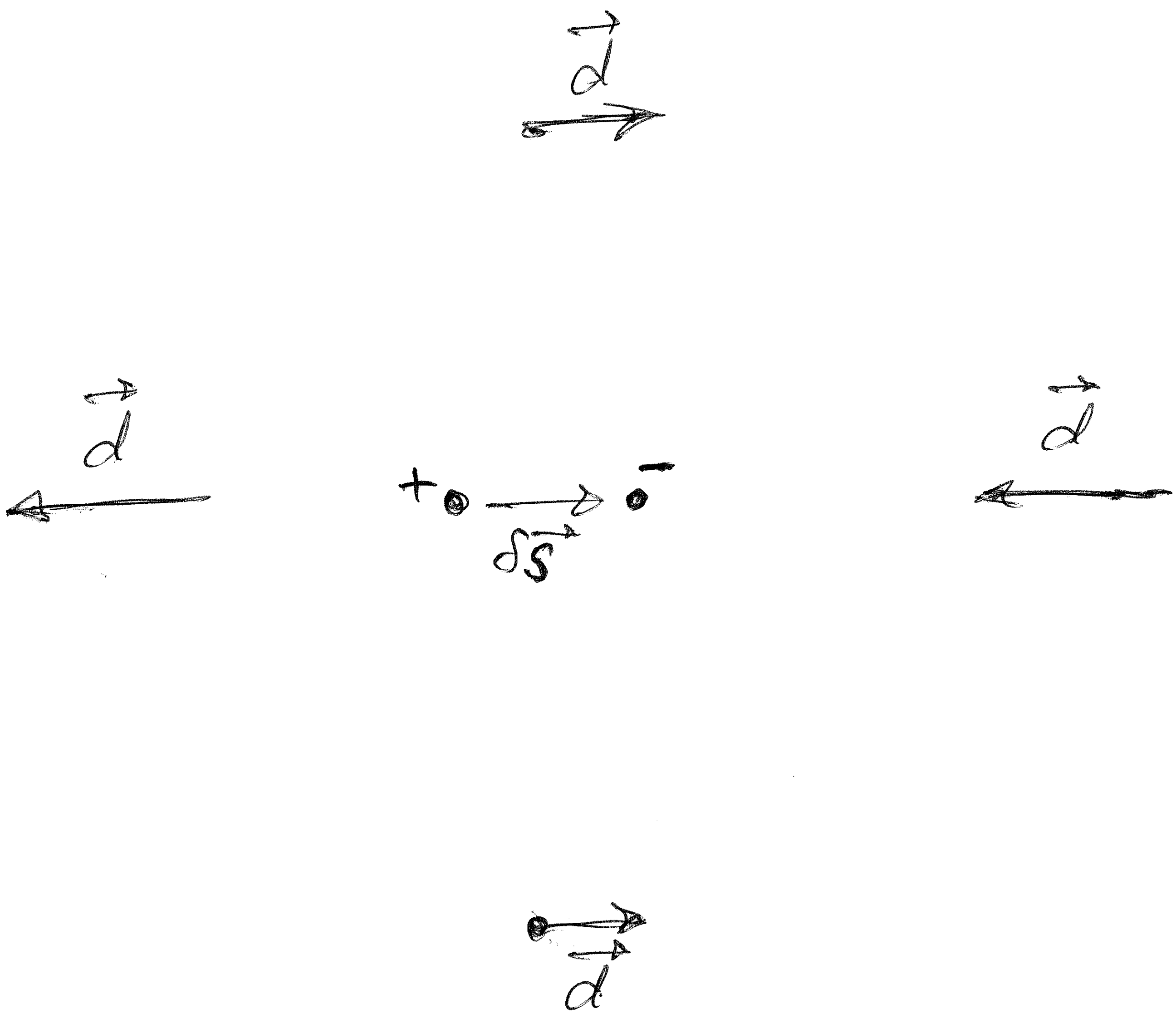
$$= \left(\frac{kq}{r^3}\right)\left(\delta\vec{S} - 3(\delta\vec{S}\cdot\hat{r})\,\hat{r}\right)$$

r = distance from $\pm q$ pair

$\hat{r}$ = unit vector away from pair

$\delta\vec{S}$ = separation between pair, from $+q$ to $-q$

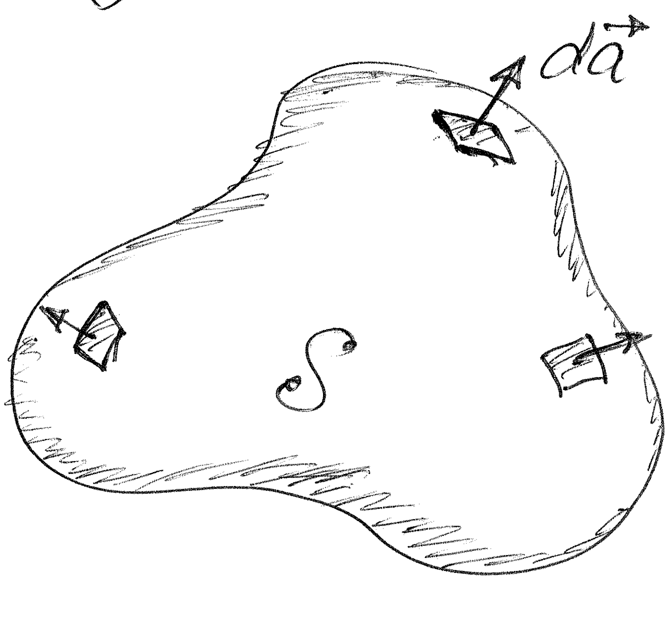
$$\vec{d} = \delta\vec{s} - 3(\delta\vec{s}\cdot\hat{r})\,\hat{r}$$



On dipole axis $\vec{d}$ is antiparallel to $\delta\vec{s}$, and $|\vec{d}| = 2|\delta\vec{s}|$; perpendicular to dipole axis $\vec{d} = \delta\vec{s}$.

The $1/r^2$ behavior of the monopole field complements the r^2 growth of the area of a sphere surrounding the monopole at radius r. Following Gauss, we ~~consider~~ define the "flux" of electric field $\vec{E}$ through a surface S by the integral

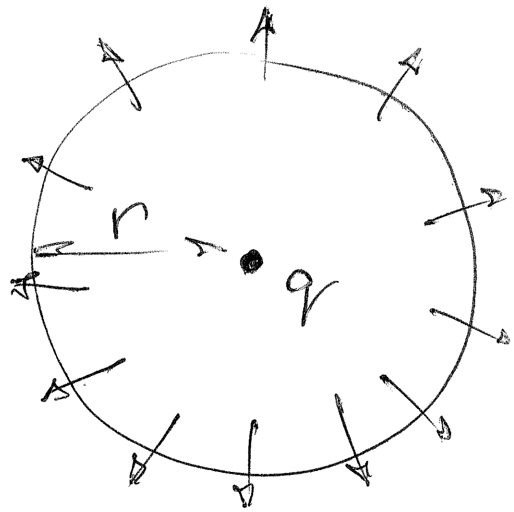
$$\Phi = \oint_S \vec{E} \cdot d\vec{a}$$



The circle on the integral sign reminds us that the definition only makes sense for surfaces that are <u>closed</u> (have no boundaries). The integral is over all infinitesimal surface elements $d\vec{a}$, whose magnitudes are the <u>areas</u> of the elements and whose directions are perpendicular to S and outward pointing. We will see that we always get the same Φ when $\vec{E}$ is that of a point charge q any where inside S. And since Φ is linear in $\vec{E}$, we always get $-\Phi$ for a charge $-q$ inside S. A dipole placed inside S gives $\Phi = 0$. This is consistent with the dipole field falling off as $1/r^3$ (if Φ is independent of $\mathcal{S}$, then we can make $\mathcal{S}$ very large but its r^2 area will not offset the $1/r^3$ decay of the dipole $\vec{E}$).

First let's work out the claimed $\mathcal{S}$-independent value of Φ for a point charge using a very symmetrical $\mathcal{S}$. Later we'll show the result holds for arbitrary $\mathcal{S}$, as long as the point charge is *inside*.

The symmetrical $\mathcal{S}$ is a sphere of radius r centered on the charge :



$$\vec{E} \text{ on } S = \frac{kq}{r^2}\hat{r}\ , \quad d\vec{a} = \hat{r}\,|d\vec{a}|$$

$$\oint_S \vec{E}\cdot d\vec{a} = \oint_{\substack{\text{sphere}\\ \text{surf.}}} \frac{kq}{r^2}|d\vec{a}|$$

$$= \frac{kq}{r^2}\underbrace{\oint |d\vec{a}|}_{4\pi r^2}$$

$$\Phi = 4\pi k q$$

The combination $4\pi K$ comes up quite a lot and ~~...~~ defines the "free-space dielectric constant"

$$\epsilon_0 = \frac{1}{4\pi K}$$

(the name of this constant is not important and refers to a property we will study later in the course).
The general form of Gauss's law reads:

$$\underbrace{\Phi = \oint_S \vec{E}\cdot d\vec{a}}_{\text{def. of flux}} = \frac{Q_{enc.}}{\epsilon_0}$$

$$\underbrace{\oint_S \vec{E}\cdot d\vec{a} = \frac{Q_{enc.}}{\epsilon_0}}_{\text{content of law}}$$

$Q_{enc.}$ is the net charge enclosed by S.

Our approach to showing Φ is independent of the shape of S will be to "morph" an arbitrary S into a sphere and show that this does not change the flux subtended by elements of solid-angle: