Lecture 38

The physics of the Maxwell equation
$$\vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0$$

was first discovered by Michael Faraday. Faraday's experiments relate to the integrated form of this law:

$$\int_S \vec{\nabla}\times\vec{E}\cdot d\vec{a} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot da$$

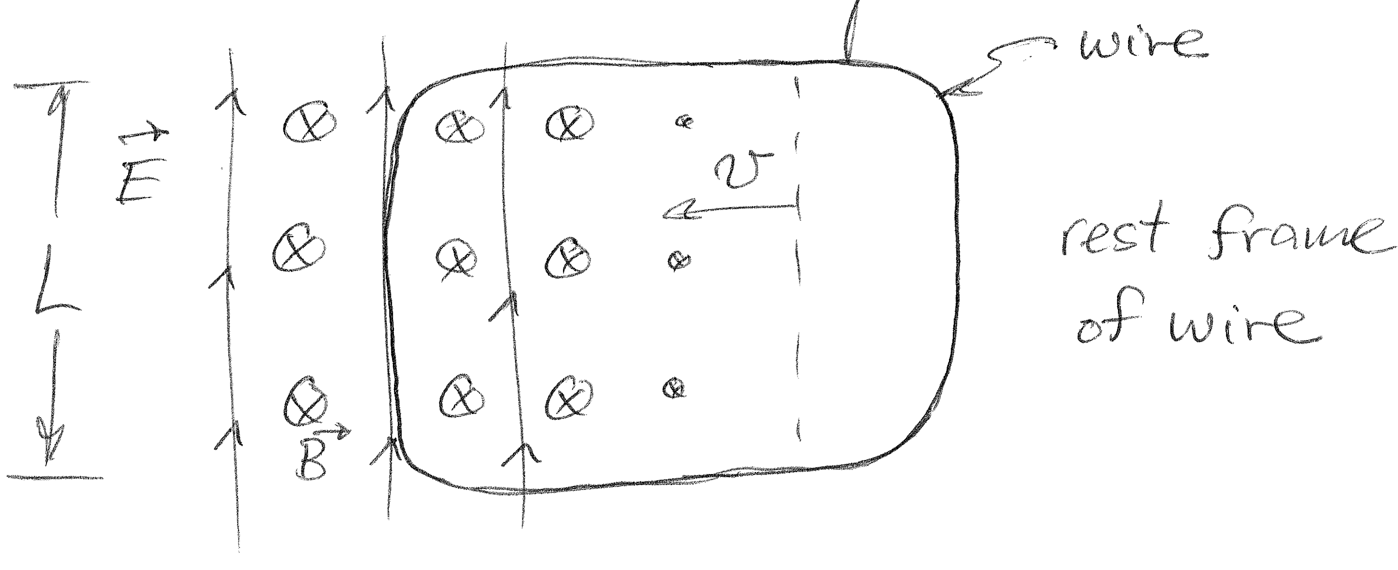
$$\Rightarrow \oint_C \vec{E}\cdot d\vec{r} = -\frac{d}{dt}\Phi_B$$ "Faraday's Law"

where S is a surface that spans

closed curve C and Φ_B is the flux of magnetic field through S:

$$\Phi_B = \int_S \vec{B} \cdot d\vec{a}$$

We first encounted a situation with changing magnetic flux in lecture 35 and we return to that scenario now. Where previously we had a curve C moving relative to the magneto-static field of a solenoid, we now have a loop of wire:



In the rest frame of the wire there is an electric field $\vec{E} = -\vec{v} \times \vec{B}$ (upward in drawing).

The circulation of $\vec{E}$ around the wire is identified with an EMF, as it has the same role a battery would have:

$$\mathcal{E} = \oint_C \vec{E} \cdot d\vec{r}$$

$\mathcal{E}$ is positive when C is oriented clockwise, and has magnitude

$$\mathcal{E} = E \cdot L = vBL = \left| \frac{d}{dt} \Phi_B \right| .$$

The sign of $\frac{d}{dt} \Phi_B$ is actually negative ($d\vec{a}$ is into the page) in agreement with Faraday's law.

Inserting some round numbers,

$$v = 1\,m/s\ ,\ B = 1G = 10^{-4}T$$

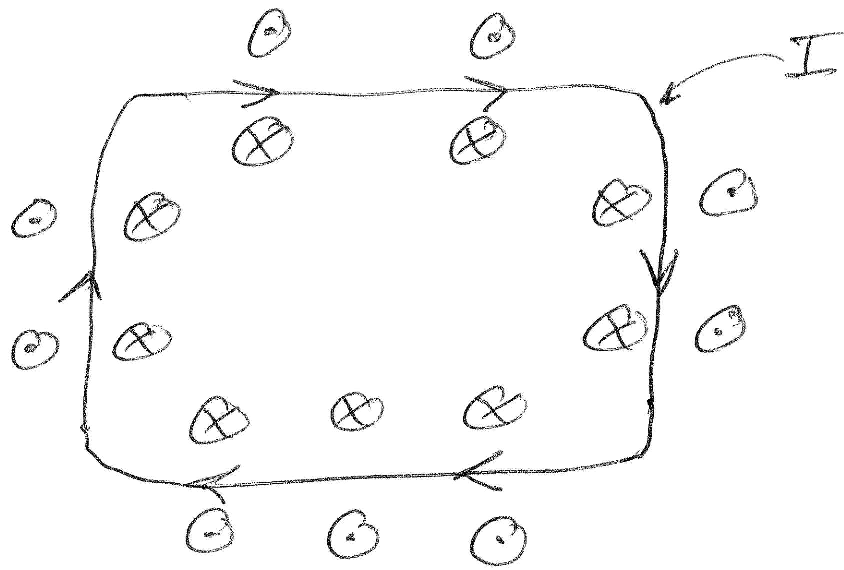
$$L = 1\,m$$

we get $\mathcal{E} = 10^{-4}$ Volt. A loop of side $L = 1m$ is large and unpractical, but we would get the same EMF using multiple coils of a smaller loop. We can understand this either as the electric field in the rest frame of the coil acting along multiple lengths of wire, or equivalently, a multiplication of the magnetic flux by the number of coils.

Some interesting physics ~~when~~ happens when the EMF starts driving a current around the wire. Suppose the total wire resistance is R, then there will be a clockwise current $I = \mathcal{E}/R$.

Heinrich Lenz came up with a nice rule for working out the orientation of the EMF based on the direction of this current. As you know, the current in the wire will create a magnetic field of its own.

Lenz's rule is that the direction of the current is such that the magnetic field it produces has the effect of <u>opposing the changing flux</u>. In our example, the flux ⊗ into the page is decreasing with time, so Lenz's current will try to introduce new flux ⊗ as shown below:

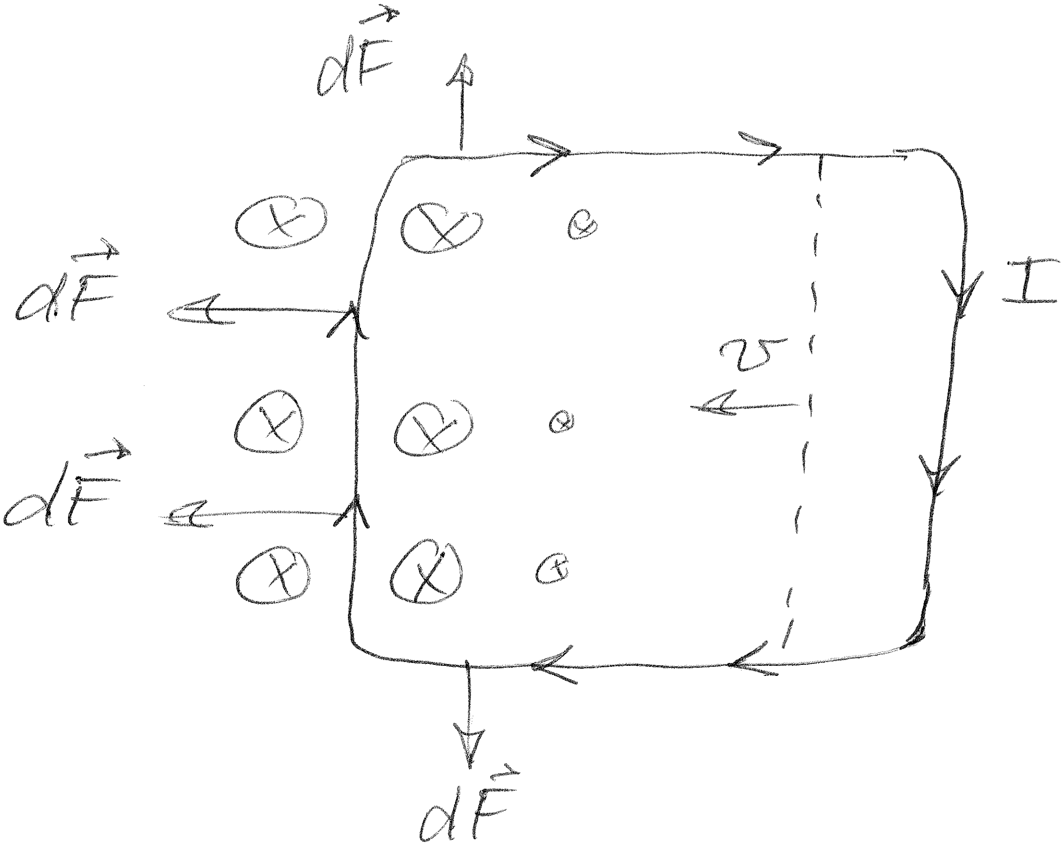


The "induced" current, by Lenz's law, tries to maintain the status quo — the original magnetic flux through its own magnetic field.

Now you also already know that a current-carrying wire will experience a force in the presence of a magnetic field. Here we are again referring to the magneto-static field the loop is in motion with respect to. This field appears in two roles: (1) its changing flux through the loop drives the current and (2) applies a force to the loop once the current is flowing. For the following calculation, recall that the force on an element of wire $d\vec{r}$ carrying current

I is:

$$d\vec{F} = I\, d\vec{r} \times \vec{B}$$



$d\vec{r}$ = in direction of positive current

The net force is in the same direction the magneto-static field is moving, i.e. it acts to reduce the relative velocity. Let's calculate the magnitude of the net force for a rectangular loop of width L:

$$F = ILB = (\mathcal{E}/R)LB$$

$$= \left(\frac{vBL}{R}\right)LB = \frac{(BL)^2}{R}v$$

We can write this as a vector equation like this

$$\vec{F} = -\frac{(BL)^2}{R}\vec{v}$$

where $\vec{v}$ is now the velocity of the loop relative to the magnetostatic field. This has the form of a frictional force. The mechanical energy always decreases because $\vec{F}\cdot\vec{v}$ is negative, which is not surprising given the resistance R.