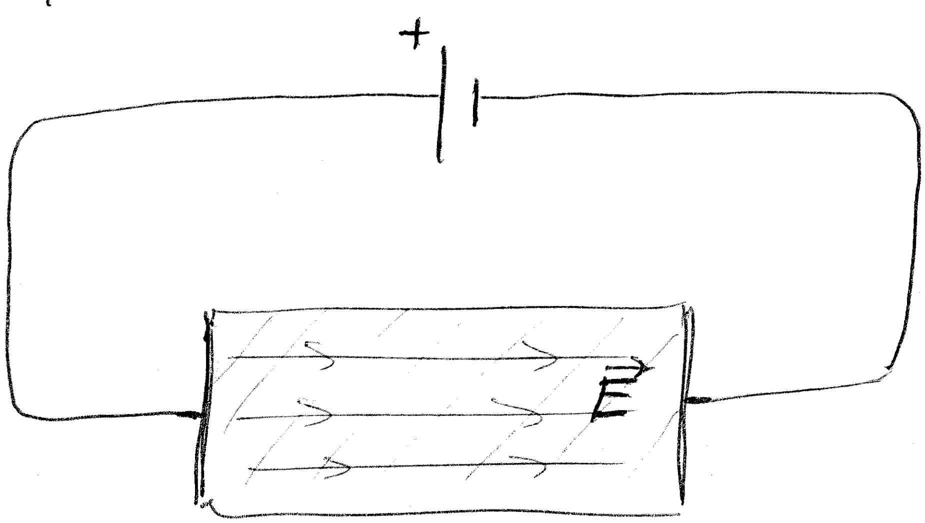
Lecture 32

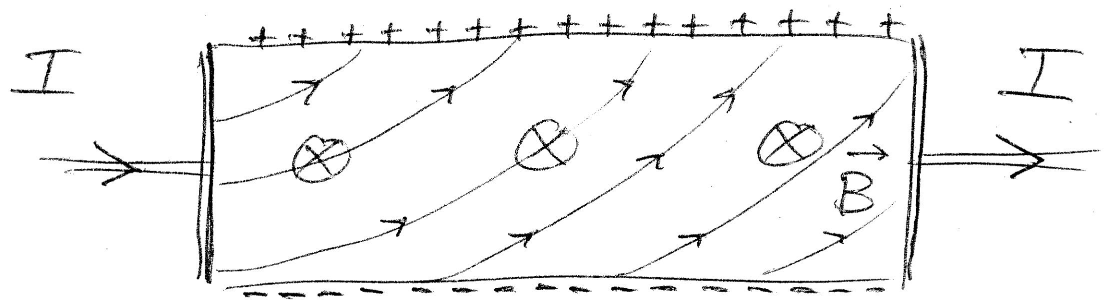
In this lecture we will take a closer look at the interplay of electric and magnetic forces when there is a steady current flowing in a conductor.

Suppose we have a slab of conducting material connected to a battery like this:



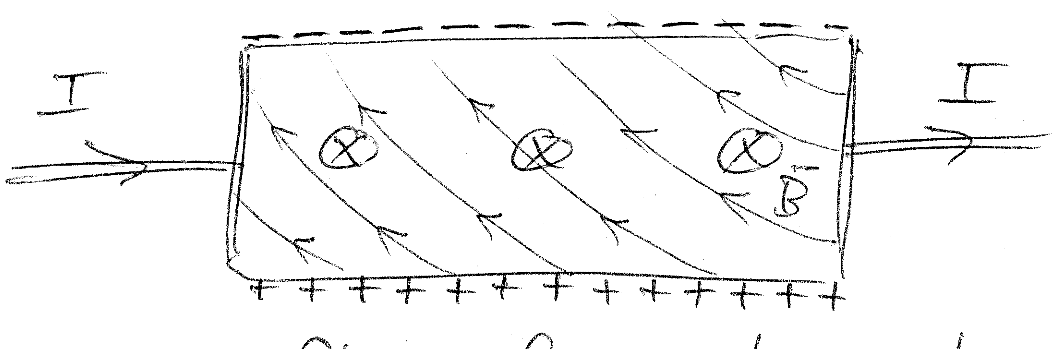
We have deliberately expanded the width of the slab because we are interested in what happens in its interior. Our analysis applies to any conductor, no matter how thin (e.g. wires).

With just the battery, positive free charges in the conductor will drift in the direction of $\vec{E}$, reaching a steady-state velocity when the electric force is equal and opposite to the friction force. Now add a uniform <u>magnetic</u> <u>field</u> whose direction is into the slab. This will deflect the free charges and cause them to accumulate along the edges of the slab:



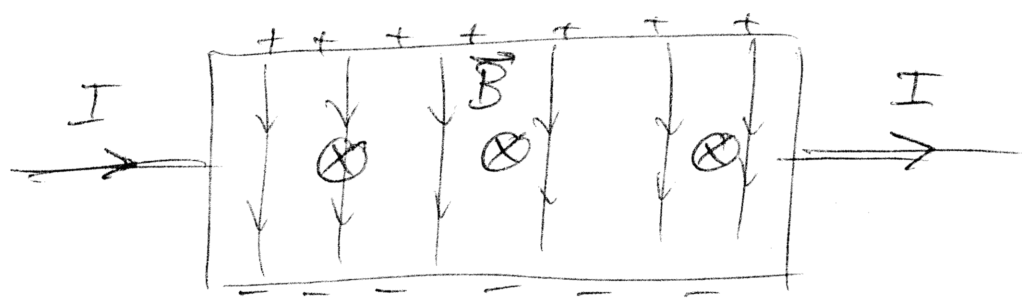
flow of positive charge
in presence of magnetic field
(lower edge is left with
net negative charge)

We have the opposite polarization
of the slab edges when the
positive current I is due to
negative free charges:

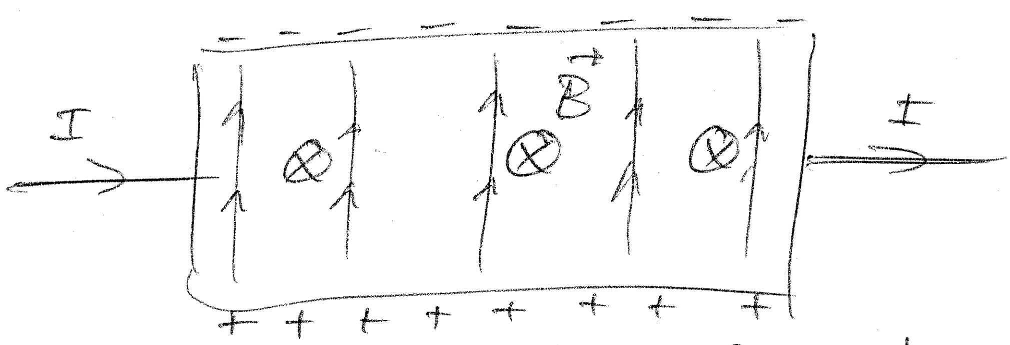


flow of negative charge

As the charge builds up along the edges, so will the electric field produced by this charge. Eventually, there will be a new steady-state where the magnetic force deflection toward the upper edge is balanced by the electric force toward the lower edge:

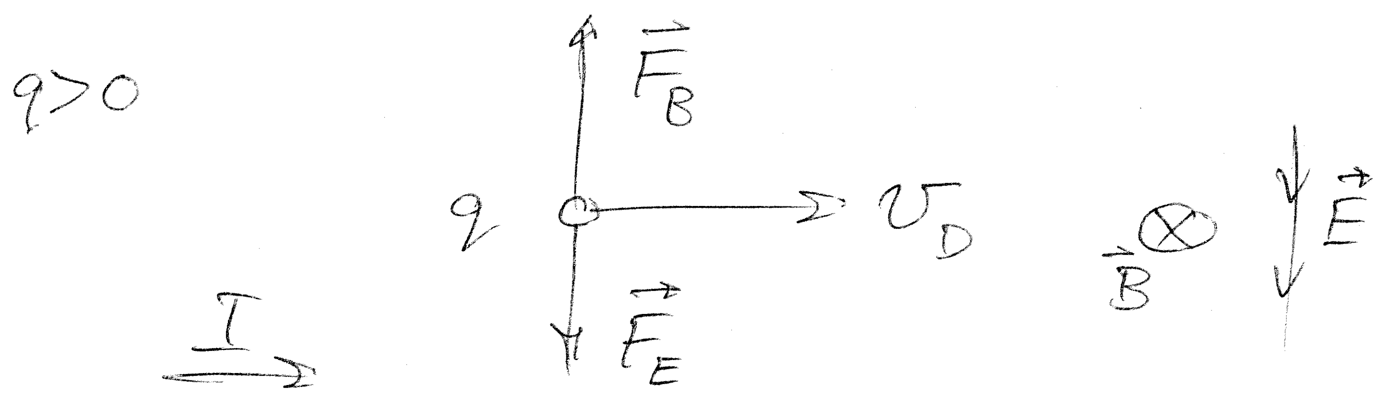


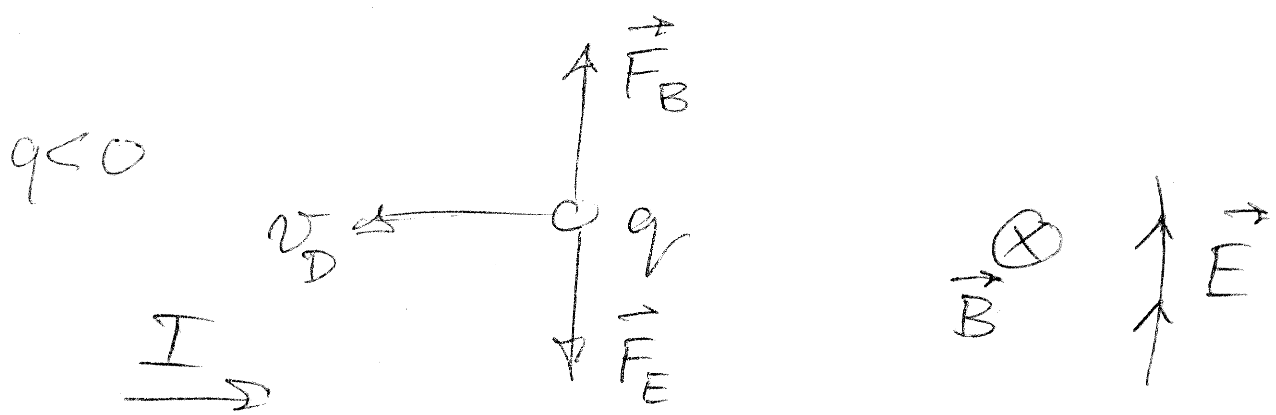
positive free charge



negative free charge

The vertical electric field $\vec{E}$ is oriented oppositely, for the two signs of charges, even though the electric force is always down:





In both cases positive current flows to the right and $\vec{B}$ is into the page. So from the polarity of $\vec{E}$ we can infer the <u>sign</u> of q (the free charges). This is called the "Hall effect".

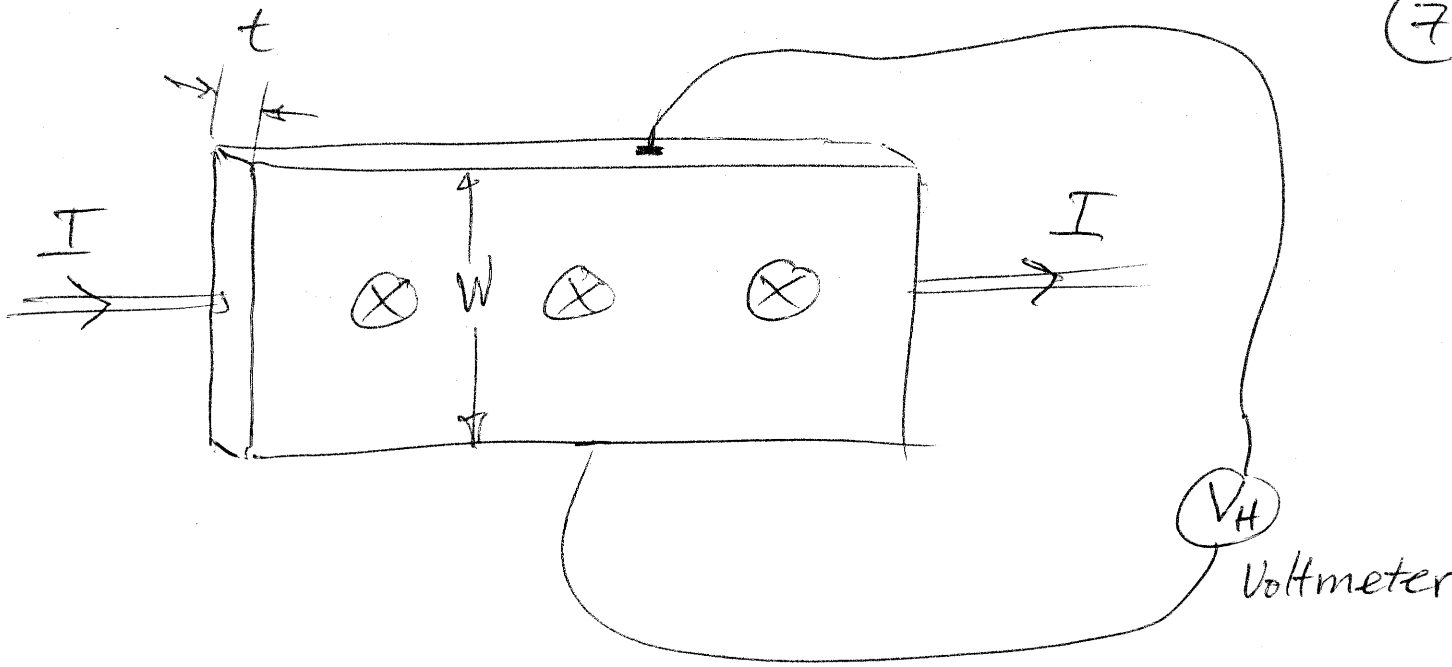
In the steady-state we have

$$|\vec{F}_E| = |\vec{F}_B|$$

$$qE = qv_D B$$

$$\Longrightarrow \quad E = v_D B.$$

A device called the "Hall probe" makes use of this relationship to measure B by measuring E. The product $E \times$ (width of slab) gives the Hall voltage V_H, which is what is actually measured:



$$I = jA = (q n v_D)(t w) \qquad (1)$$

$$V_H = E \cdot W = v_D B \cdot w \qquad (2)$$

from (1) : $\quad v_D W = \dfrac{I}{qnt}$

substitute in (2) :

$$V_H = \left(\frac{I}{qnt}\right) B$$

The sign of V_H tells us the sign of q. If we keep the current I constant, then

$\left(\frac{I}{qnt}\right)$ is a fixed proportionality between V_H and B that we can use to measure B.

Since qE and qvB are both force, the ~~MKS~~ MKS unit of magnetic field is

$$[B] = \frac{N}{C}\frac{s}{m} = V\frac{s}{m^2}$$

This combination is called the "Tesla", or T. Often the unit $10^{-4}\,T = G$, called "Gauss", is used. The magnitude of the earth's magnetic field, at the surface of the earth, is about $\frac{1}{2}$ G.

The physics of the Hall effect gives us a better understanding of the mechanism whereby a magnetic field imparts a force to a current-carrying wire. There are actually three forces:

$\vec{F}_B$ = magnetic force on the free charges

$\vec{F}_E$ = electric force on the free charges due to electric fields from charges bound to the wire surface

$\vec{F}_{wire}$ = force on wire due to electric force on bound charges free charges in steady state:

$$\vec{F}_B = -\vec{F}_E$$

Newton's 3rd law:

$$\vec{F}_E = -\vec{F}_{wire}$$

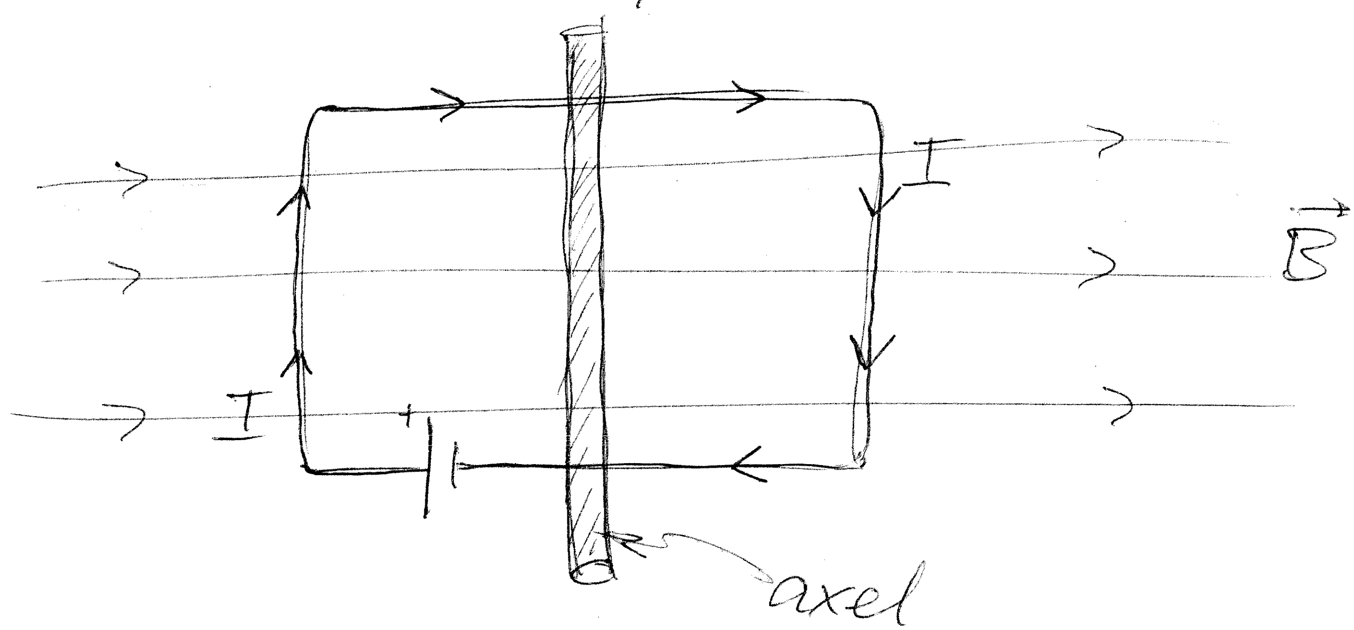
$$\Longrightarrow \quad \vec{F}_{wire} = \vec{F}_B$$

From the vector properties of the magnetic force,

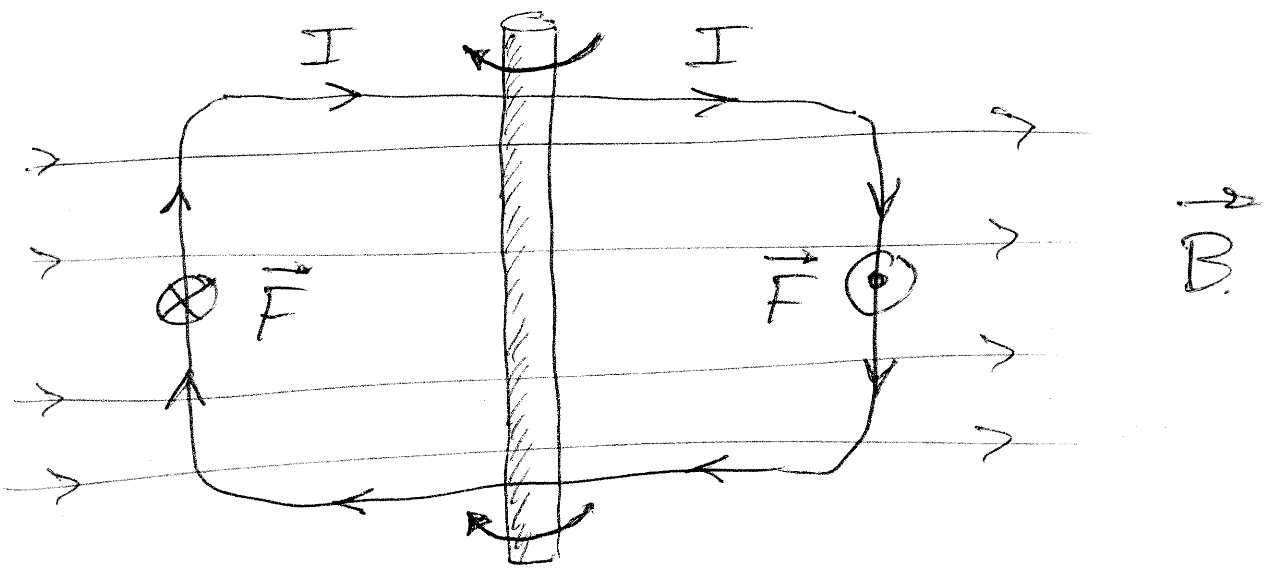
$$\vec{F} = q\,\vec{v} \times \vec{B}$$

we see that $\vec{v} \cdot \vec{F} = 0$, i.e. apparently the <u>magnetic force</u> is <u>unable</u> to <u>change the energy</u> of a mechanical system. This leads to puzzles when ~~we~~ applied to even simple systems of current-carrying wires.

Consider a rectangular loop of wire around which circulates a current powered by a built-in battery:



The loop is attached to an axel which enables it to rotate. There is also a ~~u~~uniform magnetic field; at the instant shown it is perpendicular to two sides of the loop and parallel to the other two. Shown on the diagram below are the magnetic forces on the loop ($\vec{F} = q\vec{v} \times \vec{B} \rightarrow \vec{F}_{wire} = I\vec{L} \times \vec{B}$)



So while there is no net force on the loop, there is clearly a torque causing it to turn as shown. If the loop starts at rest it will gain some rotational kinetic energy. But how is that possible if the only external agency, the magnetic field, does no work ??