Lecture 2

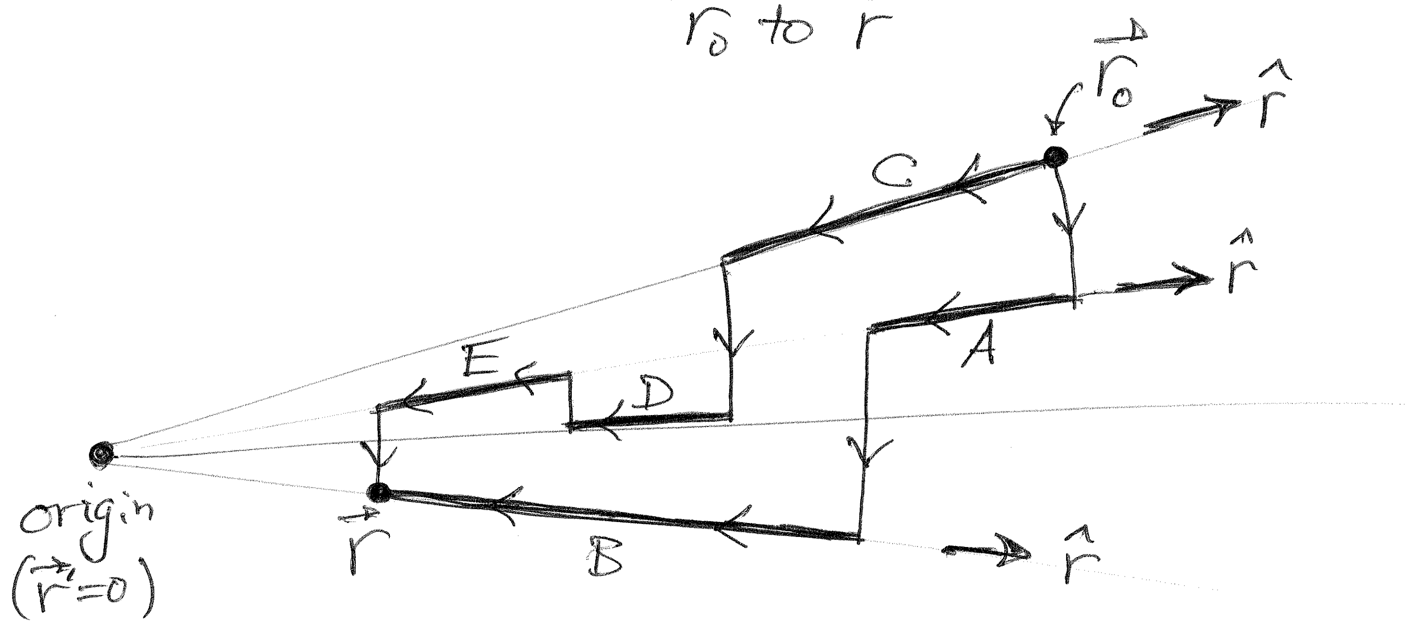
The electric force between point charges is a <u>central</u> force, and is therefore <u>conservative</u>.

Q: How does the force being central make it conservative?

A: When a force is conservative, the line integral of the force is <u>independent</u> <u>of</u> <u>path</u>, so we can consistently define a potential energy function:

fix charge 1 and move charge 2 relative to it; let $\vec{r} = \vec{r}_2 - \vec{r}_1$

$$U(\vec{r}) - U(\vec{r}_0) = - \int_{\substack{\text{path from} \\ \vec{r}_0 \text{ to } \vec{r}}} \vec{F}_{21}(\vec{r}') \cdot d\vec{r}'$$



$\vec{F}_{21}$ is (anti)parallel to $d\vec{r}'$ on pieces A, B, C, D, E and perpendicular on the other parts. Only the radial pieces (A, B, C, D, E) contribute to the line integral. And since $\vec{F}_{21}(\vec{r}')$ depends only on $|\vec{r}'|$

(the distance from charge 1), the two paths give the same result:

$$\int_{A+B} = \int_{C+D+E}$$

We still have an arbitrary additive constant in the definition of the potential energy. The usual choice, for a pair of point charges, is to set $U(\infty) = 0$ (zero potential energy when separation is infinite). This is the same choice used for the gravitational potential energy of spherical masses, and in fact the calculation of the potential energy is exactly the same because the force law is also an inverse-square central force:

$$|\vec{r}| = r$$

$$U(r) - \underbrace{U(\infty)}_{0} = -\int_{\infty}^{r} K q_1 q_2 \frac{(+dr')}{r'^2}$$

$$= +K \frac{q_1 q_2}{r}$$

To compute the potential energy of a configuration of 3 charges we start with one (charge 1) at the origin and the others at infinity. Setting the energy of this to zero, as before, we calculate the work-line-integral first for particle 2 with energy 5

$$U_{12} = K \frac{q_1 q_2}{r_{12}}$$

and then for particle 3. The latter experiences a force both from particle 1 and particle 2, which add vectorially by the principle of superposition:

$$-\int_{path} \left(\vec{F}_{31}(\vec{r}_3{}') + \vec{F}_{32}(\vec{r}_3{}') \right) \cdot d\vec{r}_3{}' =$$

$$\underbrace{-\int_{path} \vec{F}_{31}(\vec{r}_3\,')\cdot d\vec{r}_3\,'}_{U_{13}} + \underbrace{-\int_{path} \vec{F}_{32}(\vec{r}_3\,')\cdot d\vec{r}_3\,'}_{U_{23}}$$

The net energy is therefore

$$U = U_{12} + U_{13} + U_{23}$$

$$= K\frac{q_1 q_2}{r_{12}} + K\frac{q_1 q_3}{r_{13}} + K\frac{q_2 q_3}{r_{23}} \, .$$

It's clear how this generalizes to arbitrary numbers of point particles.

In many mechanical systems we can see directly how the work we perform on them is

<u>stored</u> as potential energy (compressing solid → excess energy in chemical bonds, compressing gas → increased kinetic energy of molecules). How does This work in the case of the electric force? Where is the energy stored?

We will answer this question after introducing the <u>electric</u> <u>field</u> $\vec{E}(\vec{r})$. The electric field is not just a "mathematical device" but a <u>real</u> <u>Thing</u> (the fact that you "can't see it" doesn't make it less real than a charge; for that matter, is it even possible to "see charge"?)

The key idea is that it is the electric field $\vec{E}(\vec{r})$, at the point $\vec{r}$, that produces the force on a charge at $\vec{r}$, not "some other charge far away". It's true that this electric field is created by that "other charge far away", and ~~but~~ we will see later how that happens in a local sense. For now we just want to know what kind of electric field the other particle has to create in order to be consistent with the Coulomb force. This is just a matter of judiciously re-writing the Coulomb force:

$\vec{F}_{21} = q_2 \times$ (electric field at $\vec{r}_2$ produced by charge q_1 at position $\vec{r}_1$)

$$= q_2 \, \vec{E}(\vec{r}_2 \, ; \, q_1 \, , \vec{r}_1)$$

$\vec{E}(\vec{r}; q_1, \vec{r}_1) =$ (electric field at any point $\vec{r}$, not just where we choose to put charge $\vec{r}_2$)

$$= K \frac{q_1}{|\vec{r}-\vec{r}_1|^2} \underbrace{\frac{\vec{r}-\vec{r}_1}{|\vec{r}-\vec{r}_1|}}_{\text{unit vector}}$$