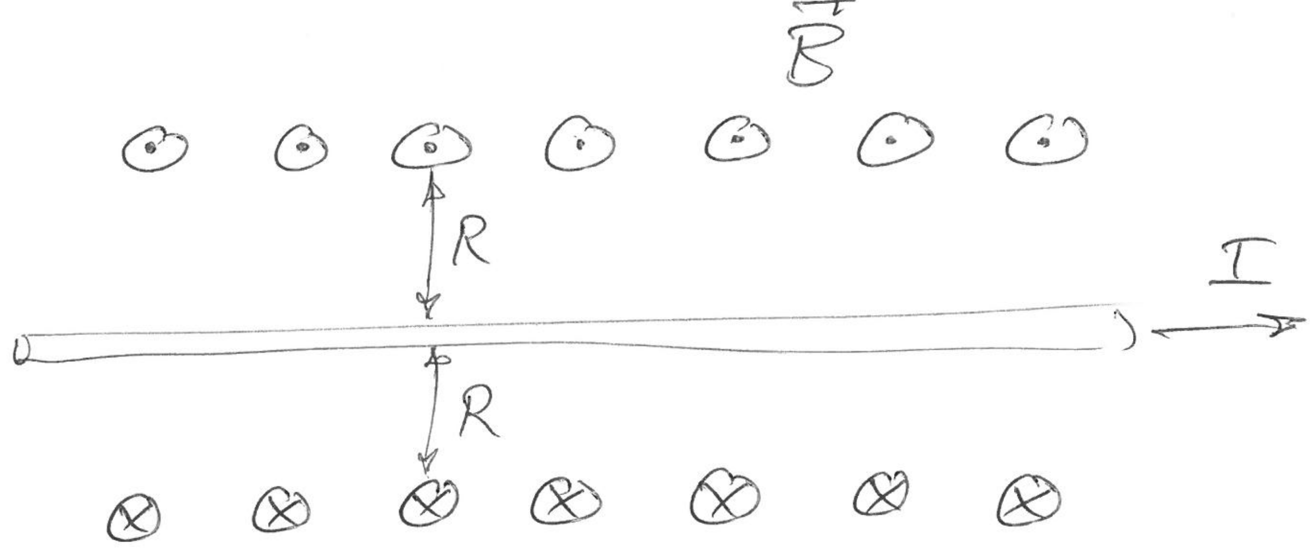
Lecture 27

We have had to go to the rest frame of the test charge to establish the existence of a force caused by currents in a wire. To explain the force in the lab frame, where there is no electric field, we postulate the existence of another field: the magnetic field $\vec{B}$. We can have a <u>single</u> $\vec{B}$ created by the current and yet three different forces — as we've seen in our previous analysis — if the force <u>depends on the velocity</u> $\vec{v}$ of the test charge in addition to the charge q. Our results in the rest frame of the test charge will be unchanged (i.e. still valid) if the magnetic force vanishes whenever $\vec{v}=0$ in our frame of reference. This fact, the fact that we found the magnitude of the force always proportional to $q|\vec{v}|$, and the peculiar directional nature of the force, are all correctly accounted for if we postulate a force

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

in whatever frame we are in. In the rest frame the force would be purely electric. In the lab frame, where $\vec{E}=0$, the force would be purely magnetic if we give $\vec{B}$ the following direction in relation to the current:



$\odot$ = out of page $\otimes$ = into page

The magnitude we determine previously:

$$B = \frac{\mu_0 I}{2\pi R}$$

There is a right-hand-rule convention in the definition of the cross product, which might lead you to the startling conclusion that the magnetic force has an intrinsic "handedness". We actually know that this is <u>not</u> the case, since we determined the direction of the force from the electric field in the rest frame and that made no reference to right-hand-rule conventions. In fact, we could just as well interpret the magnetic force law with a left-hand-rule convention for the cross product provided we reversed the direction of $\vec{B}$ in our diagram. The direction of $\vec{B}$, in relation to the direction of the current I, also requires a handed-ness convention. The choice made in the diagram is ~~called~~ the right-hand-rule for currents and the direction of the encircling $\vec{B}$ field. Were <u>both</u> this rule and the cross product rule replaced by their left-handed counterparts, the direction of the force would be unchanged.

It is often remarked, that although the electric force is very strong, in practice we witness only a feeble ghost of this force because matter is almost perfectly charge neutral. Our model of current in a wire reveals the magnetic force to be the result of special relativity (Lorentz contraction) subverting the charge neutrality of matter.

With this perspective it is less surprising that magnetic forces can be <u>strong</u> even while being just "a relativistic correction". To get a sense of magnitudes, let's estimate the force between two parallel wires, both carrying current I. We'll think of one wire being the source of $\vec{B}$, ~~with~~ and the charges flowing in the other wire as experiencing the force. There may be one or several species of charges flowing, all experiencing the magnetic force. The net current density is

$$j = n_1 q_1 v_1 + n_2 q_2 v_2 + \cdots$$

n = number density
q = charge
v = drift velocity

Consider a wire of length L and cross-sectional area A. Then

$$I = A j$$

$$L I = L A (n_1 q_1 v_1 + \cdots)$$

$$= N_1 q_1 v_1 + N_2 q_2 v_2$$

since LA is the volume of the wire and LAn_1 is the total number (N_1) of charges of type 1, etc.

Because qvB is the magnitude of the magnetic force for any charge species, the product

$$F = LIB$$

has the simple interpretation as the net force on a wire of length L carrying current I in the presence of a field $\vec{B}$ perpendicular to the wire. Writing B in terms of the current in the source wire we obtain

$$F = \frac{\mu_0}{2\pi} \frac{L I_1 I_2}{R}$$

$$= \frac{1}{2\pi\epsilon_0 c^2} \frac{L I_1 I_2}{R}$$

We've added subscripts on the currents to help us recall that one is acting as the source of $\vec{B}$ while the other feels the force. However, the force is symmetric in the "current identities", as it should be if the source wire is going to feel an equal (and opposite) force}.

Now if $I_1 = I_2 = 1$ Amp, and

$l = R = 1$ meter, then

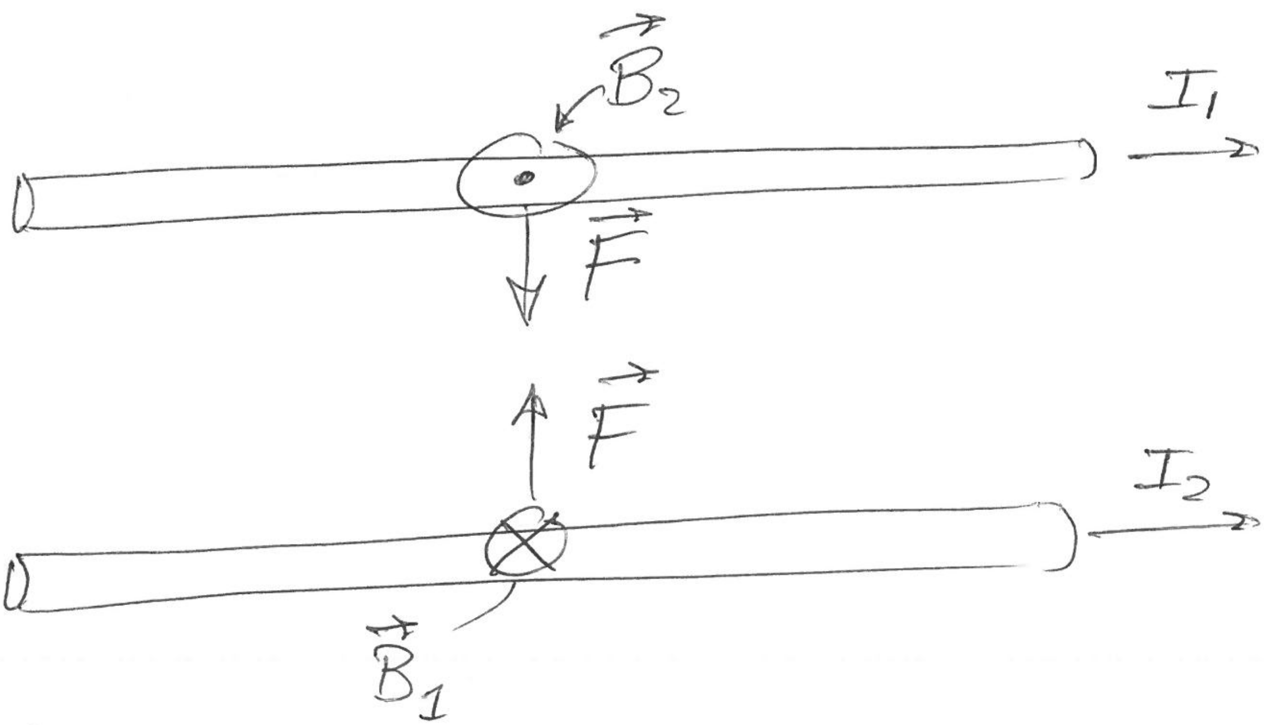
$$\left(\frac{1}{4\pi\epsilon_0} = K\right)$$

$$F = \frac{2K}{c^2} \cdot 1$$

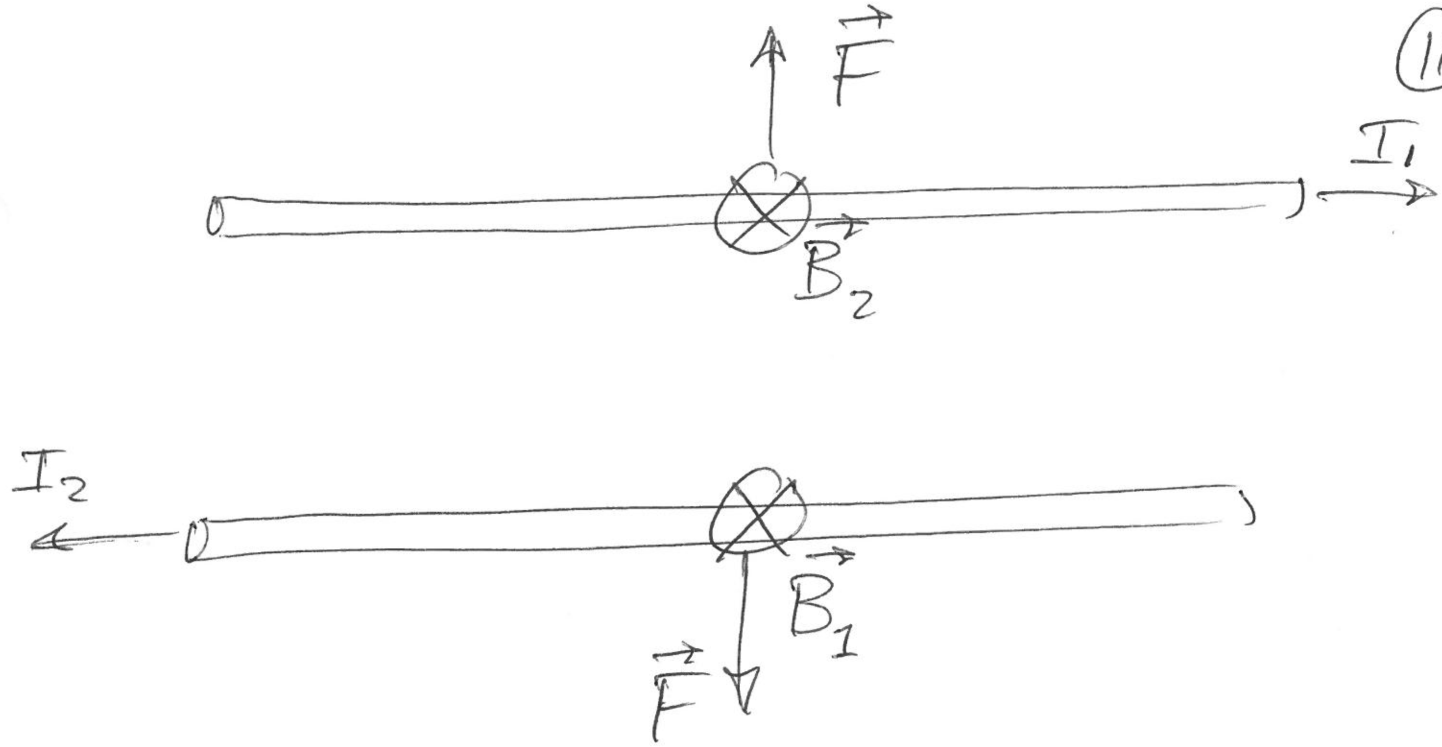
$$\simeq 2\frac{10^{10}}{(3\times10^8)^2} \cong 2\times10^{-7}\,N$$

We can increase this by a factor 10^4 just by decreasing the wire separation R to 1 cm and increasing the currents to 100 Amp. A force of 2×10^{-3} Newtons is easily measureable.

The diagrams below show the directions of $\vec{B}$ and $\vec{F}$ for both ways the currents can flow:



($\vec{B}_1$ = field due to current I_1)



Rule : "like" currents attract
"opposite" currents repel