Lecture 26

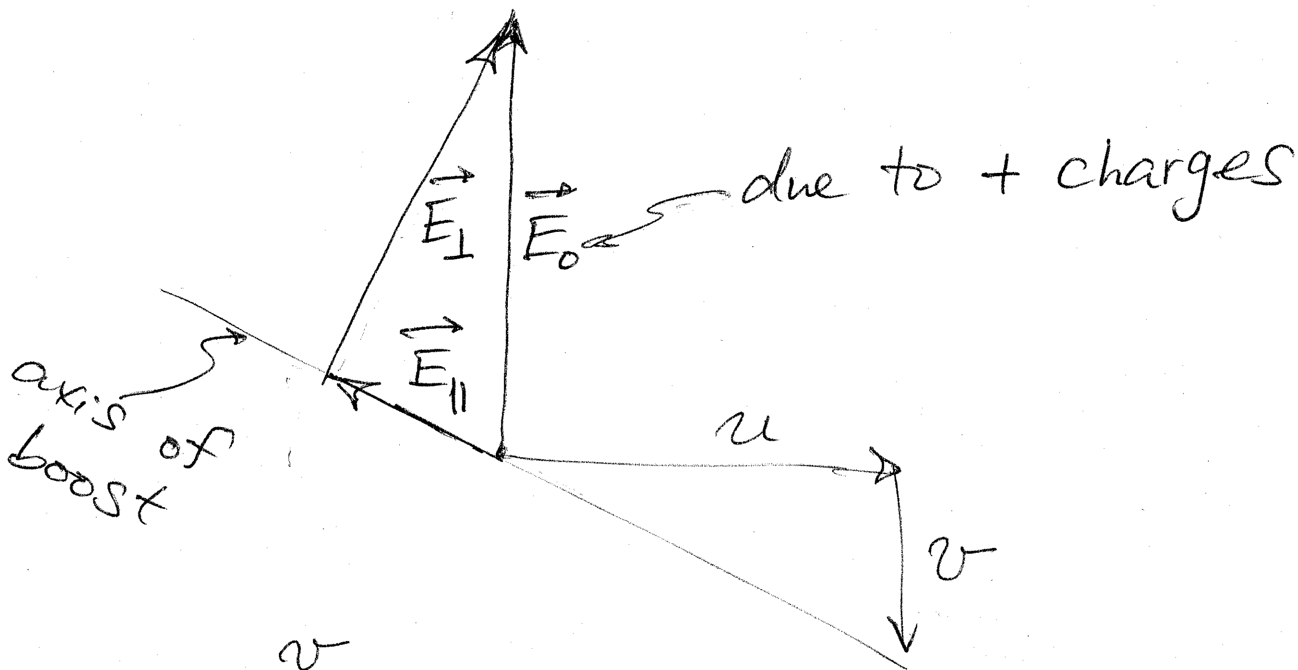
(continuation of lecture 25)

The second of these is a very counter-intuitive property of Lorentz-transformations. For small u and v the angle of rotation is approximately

$$\theta = \frac{1}{2} \frac{uv}{c^2}$$

– clearly a relativistic effect. We will ~~e~~ work out the effects of (1) and (2) in turn, assuming u and v are small (compared with c) for both.

For effect (1) we need to decompose the electric field of the line charges at rest into components parallel and perpendicular to the vector boost velocity:



$$E_{\parallel} = \frac{v}{\sqrt{u^2+v^2}} E_0$$

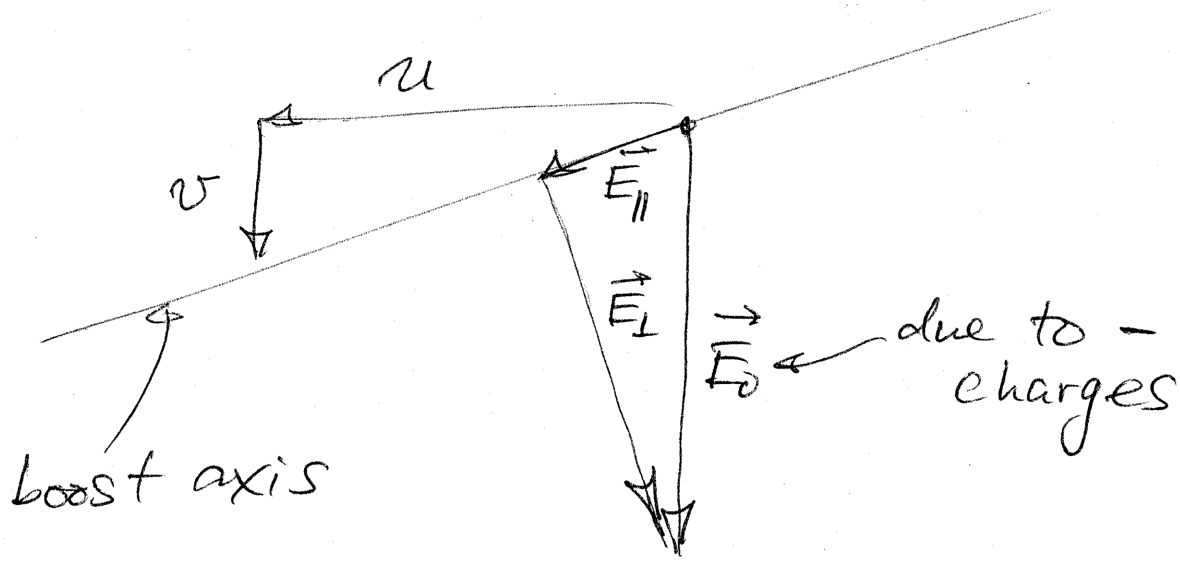
$$E_{\perp} = \frac{u}{\sqrt{u^2+v^2}} E_0$$

by similar triangles

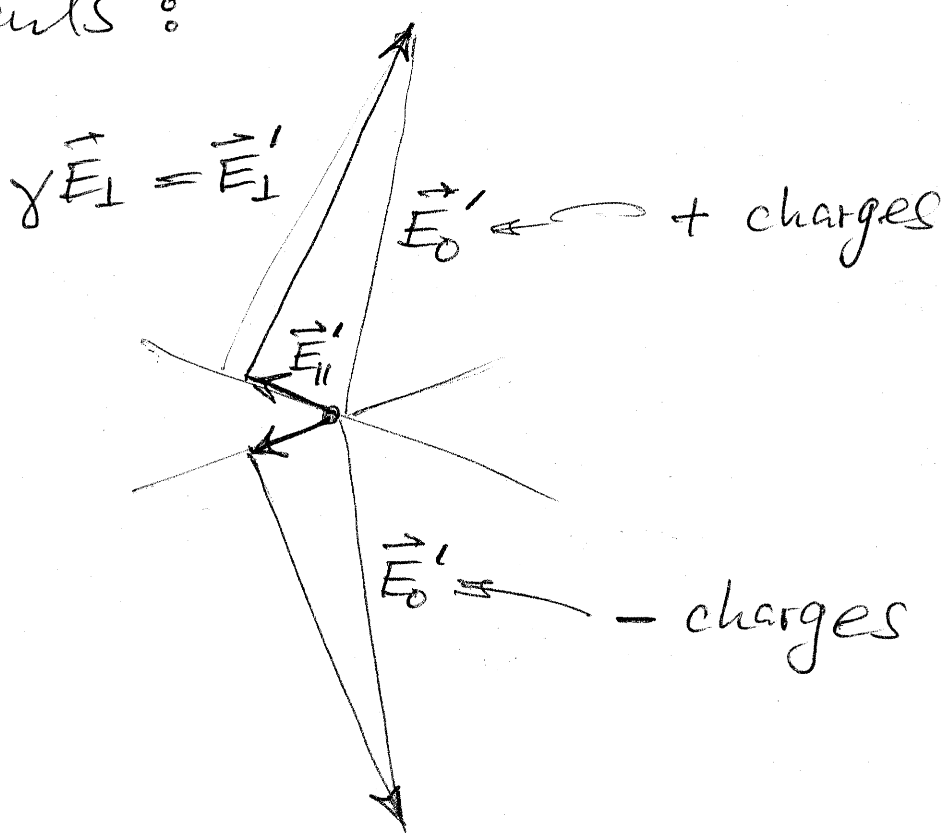
$\vec{E}_\perp$ gets enhanced by the γ factor for net velocity which has speed $\sqrt{u^2+v^2}$:

$$\gamma \simeq 1 + \frac{1}{2}\left(\frac{u^2+v^2}{c^2}\right)$$

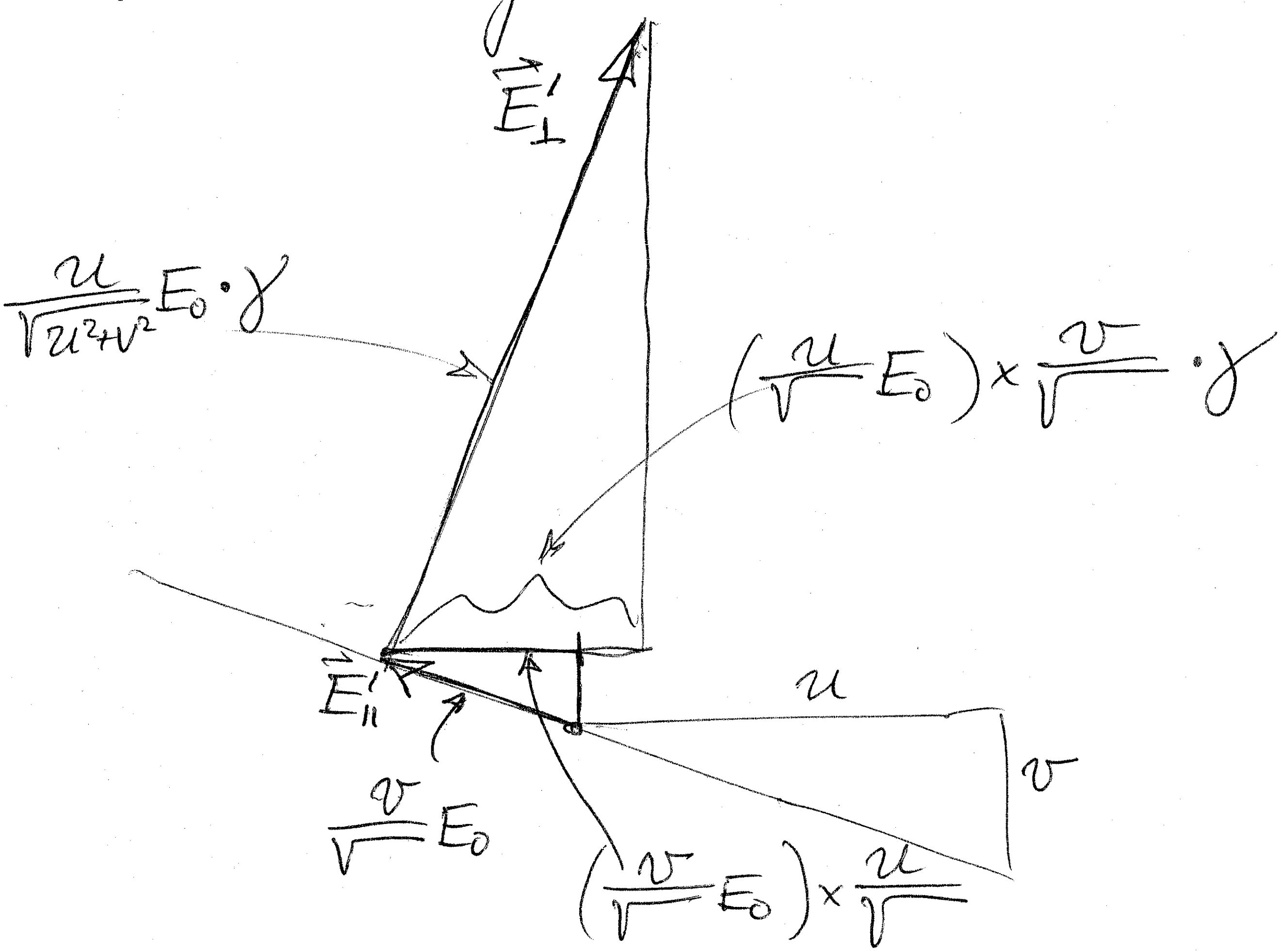
The picture for the negative charges is similar :



Here's a diagram that combines both fields, showing the enhancement of the perpendicular components:



Notice that the <u>net</u> electric field (from + and − charges combined) is a field to the <u>right</u>. We calculate the magnitude by projecting both $\vec{E}_{\parallel}'$ and $\vec{E}_{\perp}'$ onto the horizontal axis by multiplying with factors deduced from similar triangles:



$$\begin{pmatrix}\text{horizontal comp.}\\ \text{of } \vec{E}'_{\parallel} + \vec{E}'_{\perp}\end{pmatrix} =$$

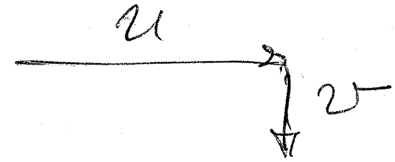
$$\left(\frac{uv}{u^2+v^2}\right) E_0 \gamma - \left(\frac{vu}{u^2+v^2}\right) E_0$$

$$= \left(\frac{uv}{u^2+v^2}\right) E_0 \ \frac{1}{2}\left(\frac{u^2+v^2}{c^2}\right)$$

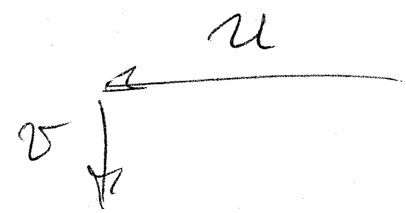
$$= \frac{1}{2}\frac{uv}{c^2} E_0$$

The negative charges ~~ex~~ contribute exactly the same amount, ~~&~~ for a total $\frac{uv}{c^2} E_0$

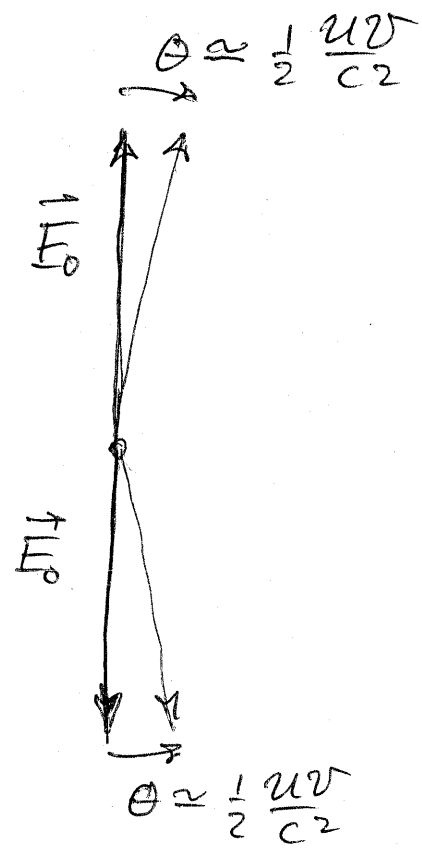
Effect (2) is also of order $\frac{uv}{c^2}$, so we can apply the effect of frame rotation without the γ enhancement of effect (1). The rotation for



and



have opposite sign with the following result:



The sum of the two rotated fields is again a field to the right; its magnitude is:

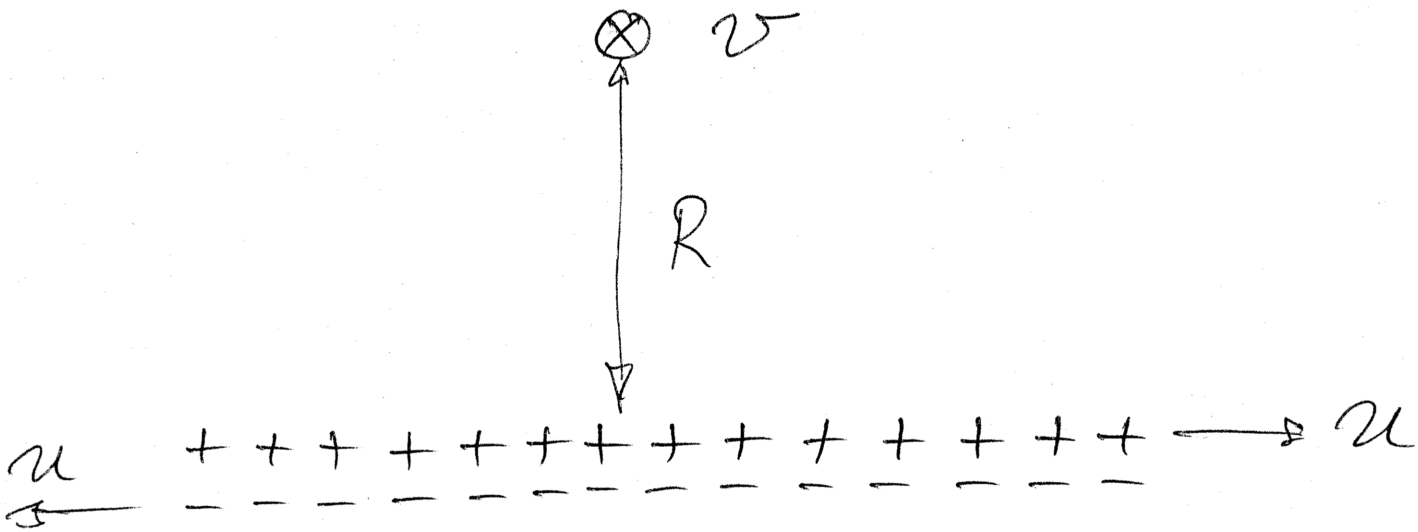
$$2 E_0 \sin\theta \simeq \frac{uv}{c^2} E_0$$

Combining effects (1) and (2) we get an electric field to the right with magnitude

$$2 \frac{uv}{c^2} E_0$$

But this is exactly the same magnitude we found when the test charge was moving to the right (and the force was down).

Let's now look at the third of the three orthogonal directions the test charge might move, and this will be the easiest!



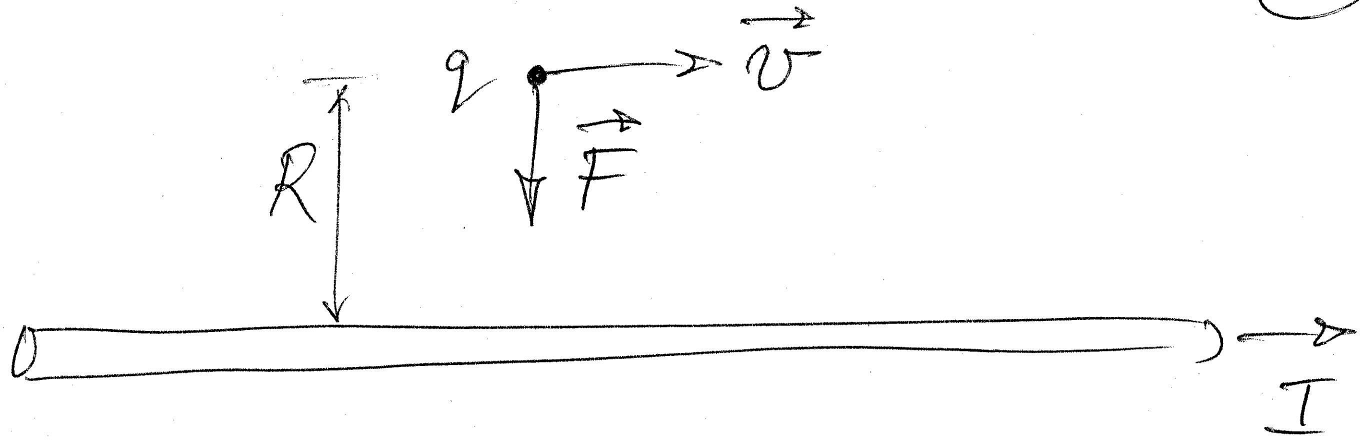
The ⊗ symbol indicates the test charge velocity is into the page. Like the first case, here the relative motion between line charges and test charge is perpendicular to the electric field. However, unlike the first case the relative speeds for the + line and − line are the same, giving equal enhancement factors. The two fields $\uparrow E_0\gamma$ and $\downarrow E_0\gamma$ will therefore cancel.

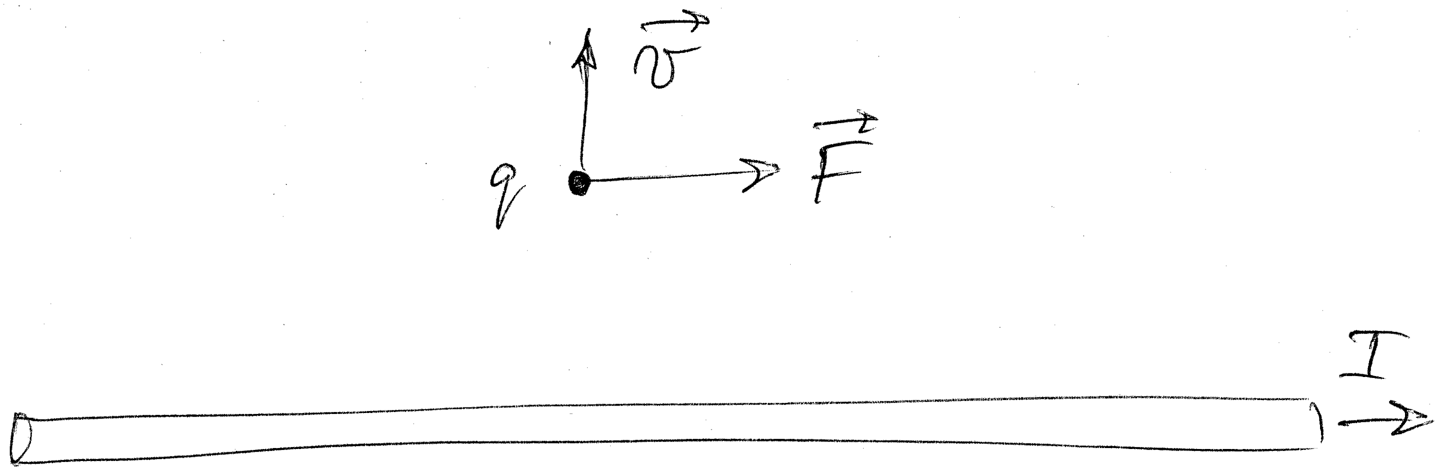
Also, like the second case we looked at, the composition of the two non-collinear boosts produces a rotation by $\frac{1}{2}\frac{uv}{c^2}$. But unlike the second case, now the axis of rotation is parallel to the electric field and so the cancellation of $\uparrow E_0$ and $\downarrow E_0$ is unaffected by that. The net electric force on the test charge for this choice of velocity is therefore zero.

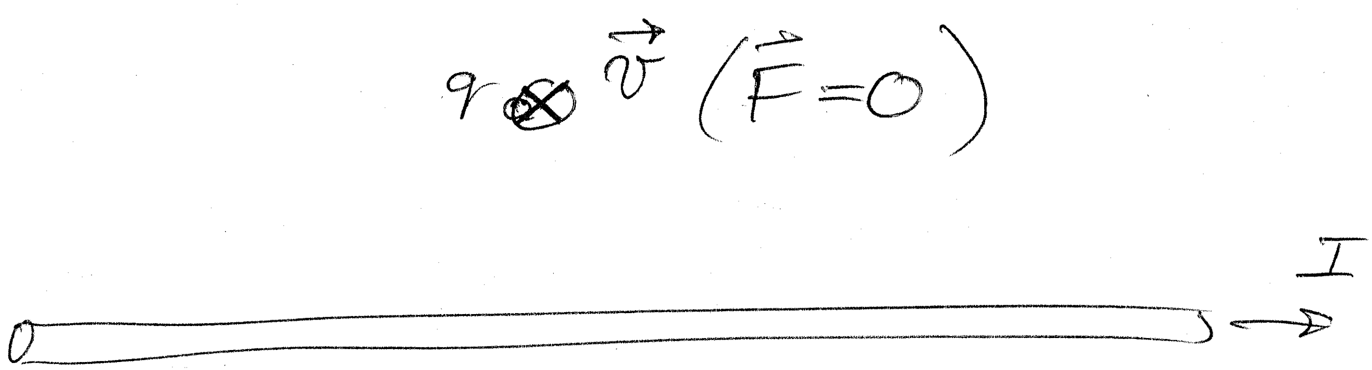
Let's summarize what we've found about the force on a <u>moving</u> test charge in the presence of a current flowing in a straight wire. Although we calculated these forces ~~at~~ in the rest frame of the test charge, we can transform them back to the "lab" frame using

$$\frac{d\vec{P}_\perp}{dt} = \frac{1}{\gamma}\frac{d\vec{P}_\perp'}{dt'}$$ (primed: instantaneous rest frame)

since in all cases the force was perpendicular to the test charge velocity. Also, since our calculations were valid only to lowest order in $(v/c)^2$, we can replace γ by 1, so $\vec{F} \simeq \vec{F}'$.


q
$\vec{v}$
$\vec{F}$
R
I


$\vec{v}$
q
$\vec{F}$
I


q $\vec{v}$ ($\vec{F}=0$)
I

$$|\vec{F}| = q v B$$

$$B = \frac{\mu_0 I}{2\pi R}$$

$$\mu_0 = \frac{1}{\epsilon_0 c^2}$$

These apply in both cases where the force $\vec{F}$ is non zero.