Lecture 25

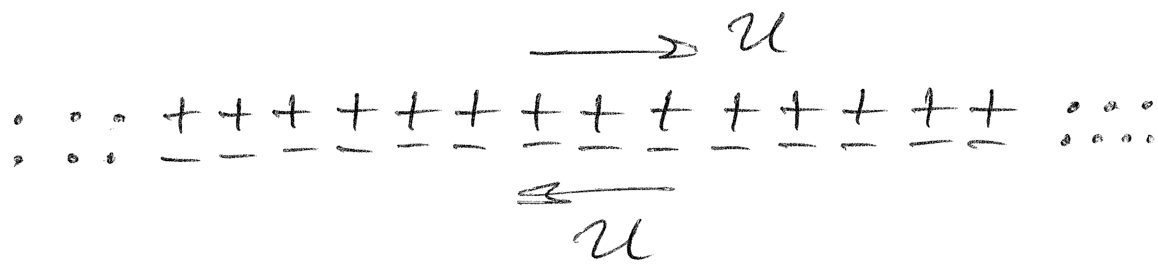
In this lecture we exploit the laws of special relativity to argue for the existence of yet another field to account for the forces on charges: the <u>magnetic</u> <u>field</u>. Using a highly symmetric representation of current as moving lines of charge, we will learn several interesting things about the magnetic field:

- the magnetic force attributed to the magnetic field in one frame is an electric force due to the electric field in another frame.

- the strength of the magnetic force emerges as a $(v/c)^2$ "correction" to the electric force

- the magnetic force has peculiar vectorial characteristics

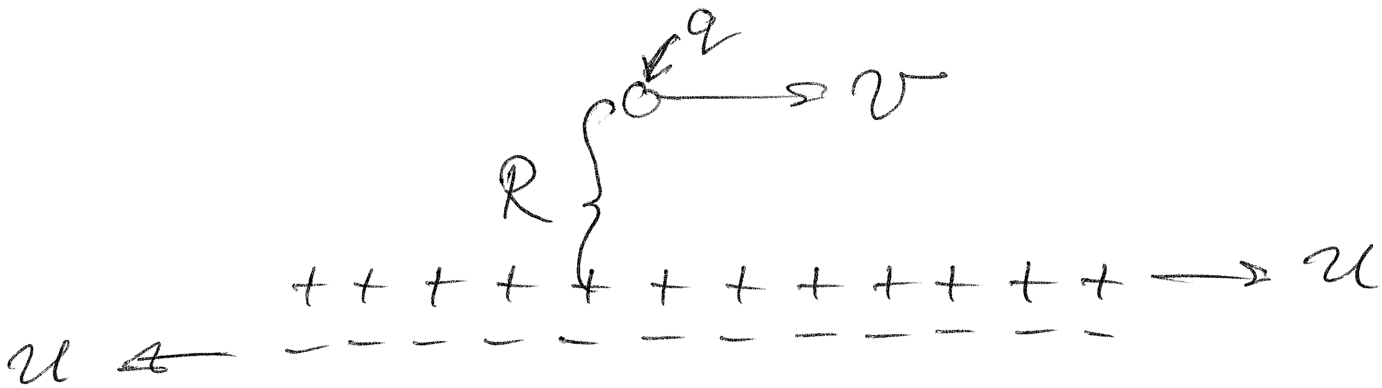
Consider a "wire" containing zero net charge but wherein two equal and opposite line densities move in opposite directions with equal speed u:



Since there is zero net charge,

there is no electric ~~ele~~ field in this frame. If a moving test charge in this frame were to experience a force, it could not be an electric force; it would have to be something else.

We will examine three ways a test charge might move relative to the wire. First consider a charge that moves parallel to the wire like this:



Using superposition, we will find the electric force on the test charge in its rest frame from moving lines of charges. Let $+\lambda$ be the linear charge density of the positive charges in their rest frame. In that same rest frame there will be a radial electric field with magnitude

$$E_0 = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{R}$$

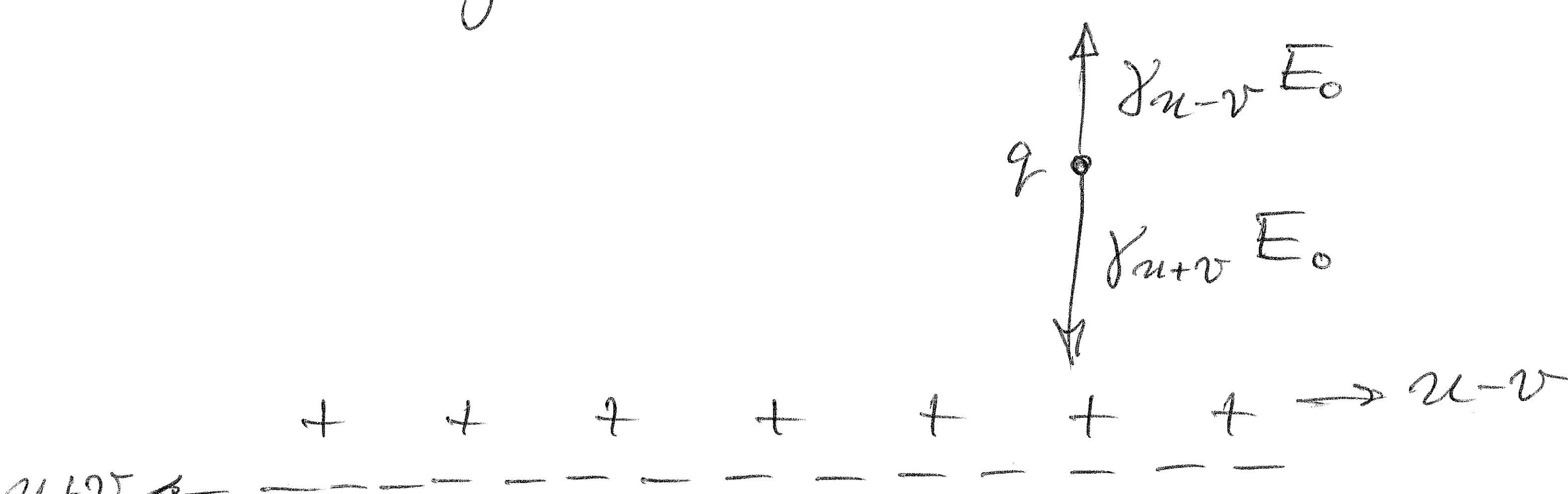
at distance R from the wire (we obtained this with Gauss's law earlier in the course). To find the electric field in the rest frame of the test charge we note that the relative motion between frames is perpendicular to the radial electric field, thereby enhancing it by the factor.

$$\gamma_{v-u} = \frac{1}{\sqrt{1-\frac{(v-u)^2}{c^2}}} \simeq 1+\frac{1}{2}\frac{(v-u)^2}{c^2}$$

The line of negative charges produces a field of the opposite sign and enhancement factor

$$\gamma_{v+u} \simeq 1+\frac{1}{2}\frac{(v+u)^2}{c^2}$$

Here is how things look in the test charge rest frame:



Since $\gamma_{u+v} > \gamma_{u-v}$, the net electric field/force is downward.

$$F = q E_{net}$$

$$= q(\gamma_{u+v} - \gamma_{u-v}) E_0$$

$$\simeq q\left(1 + \frac{1}{2}\frac{(u+v)^2}{c^2} - 1 - \frac{1}{2}\frac{(u-v)^2}{c^2}\right) E_0$$

$$= q \frac{2uv}{c^2} E_0$$

$$= q v \frac{2u\lambda}{2\pi \epsilon_0 c^2} \frac{1}{R}$$

Let's define some things to help with the interpretation. First, each line of charges (+ and −) transports charge to the right at the rate $u\lambda$, so together

$$I = 2u\lambda$$

is the current in the wire.

The counterpart of ϵ_0, for magnetic fields, is the constant

$$\mu_0 = \frac{1}{\epsilon_0 c^2} .$$

Clearly the force is proportional to q and apparently also v, the other property of the test charge. The rest defines the magnetic field B:

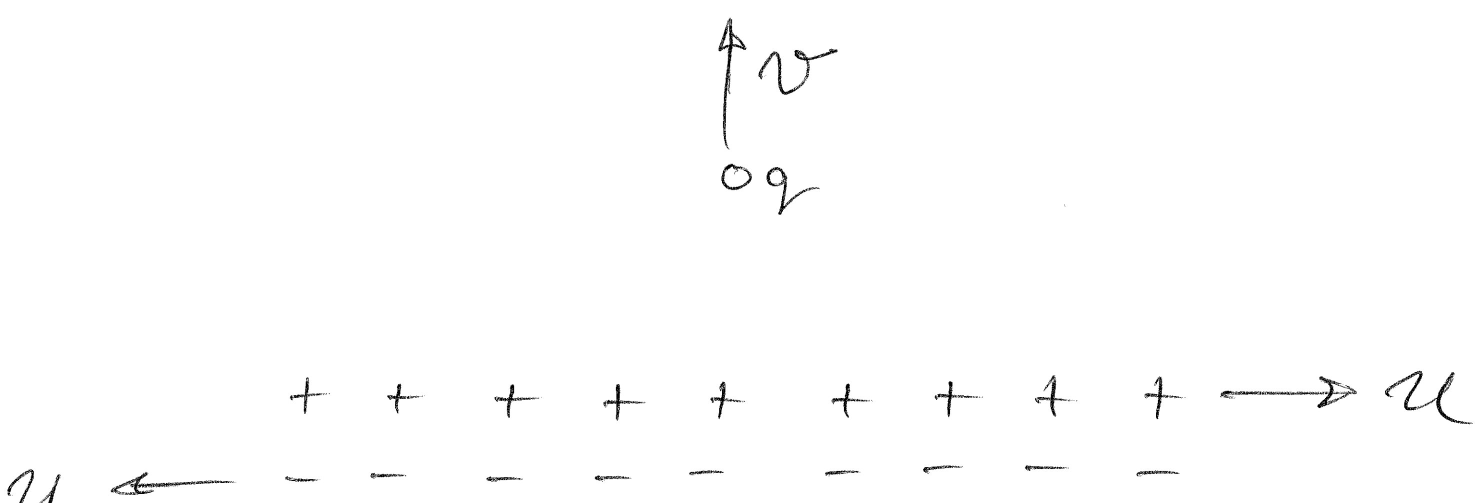
$$B = \frac{\mu_0 I}{2\pi R}$$

$$F = q v B .$$

These very same quantities will appear again, when we give the test charge a different velocity. From an earlier equation we see that

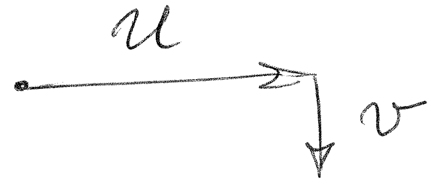
$$\begin{pmatrix}\text{magnetic}\\ \text{force}\end{pmatrix} \sim \left(\frac{vu}{c^2}\right)\begin{pmatrix}\text{electric}\\ \text{force}\end{pmatrix}.$$

Next consider the case where the test charge velocity is away from the "wire":



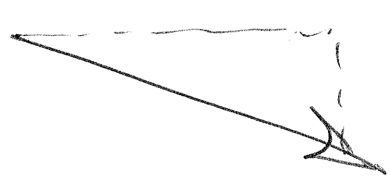
This is quite a bit more complicated to analyze than the previous case, because to get to the rest frame of the test charge we have to compose non-collinear boosts.

Starting with the electric field E_0 of the + charges in their rest frame, we first make a boost to the right with speed u, and follow that with a boost downward with speed v:



The result of combining these noncollinear boosts is two things:

(1) a boost corresponding to their vector sum



(2) a clock-wise rotation.