Lecture 23

In the last lecture we worked out an explicit formula for a point-charge electric field as seen from a moving frame ("primed"). We reproduce the formula here, omitting the prime, with the interpretation that in our (unprimed) frame the charge is moving with uniform speed v along the x-axis (we don't care that there's another frame where the charge is at rest):

$$\vec{E} = \frac{Kq}{r^2} \frac{1 - v^2/c^2}{\left(1 - v^2/c^2 \sin^2\theta\right)^{3/2}} \hat{r}$$

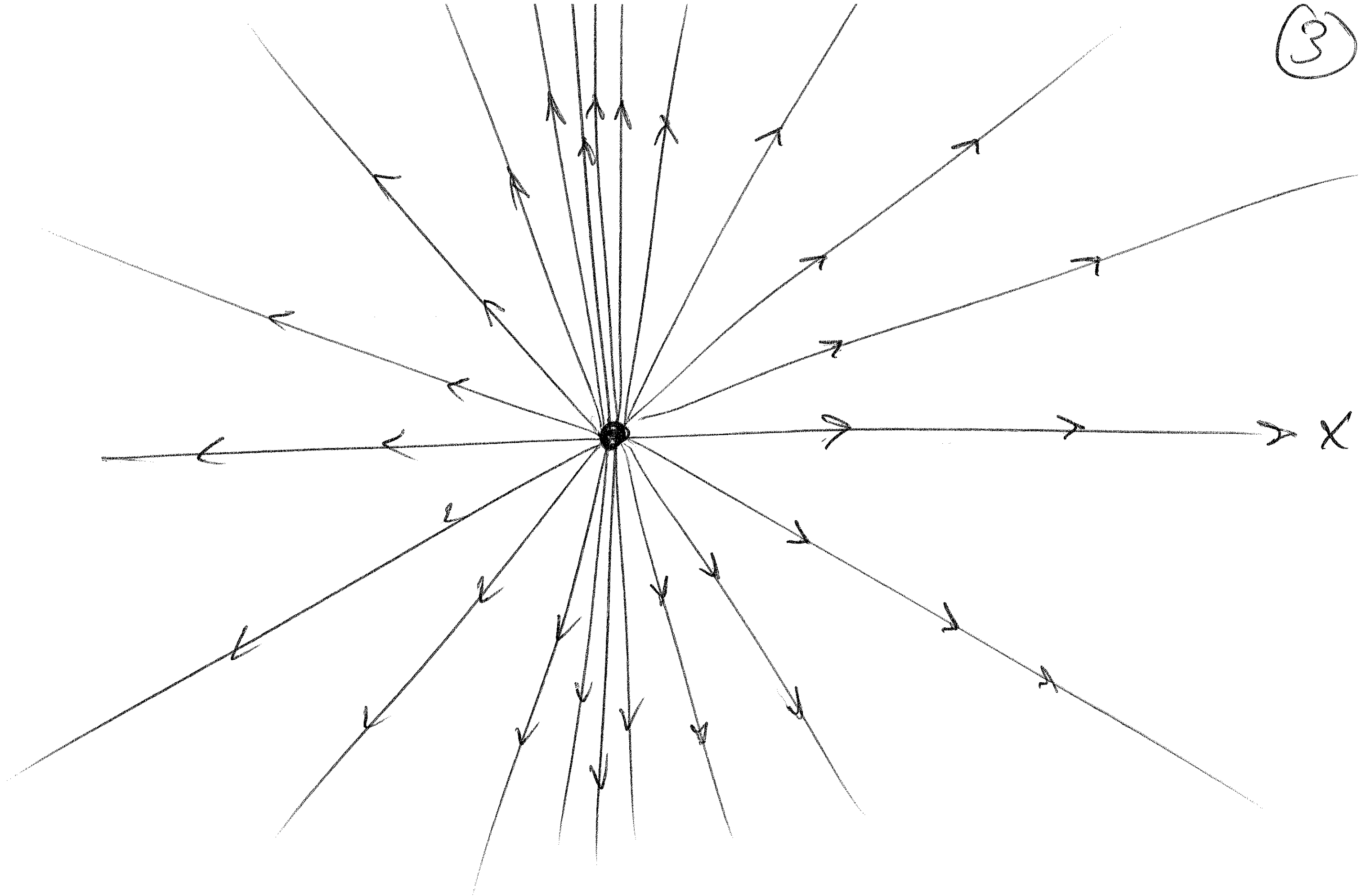
The absence of t in this formula is a casualty of our streamlined notation: t is contained _implicitly_ in r, θ, and $\hat{r}$.

r = distance to charge _at time t_

θ = angle from x-axis about the charge-origin _at time t_

$\hat{r}$ = unit vector pointing away from charge _at time t_.

Let's make a sketch of the electric field at the instant when the charge is at the origin:

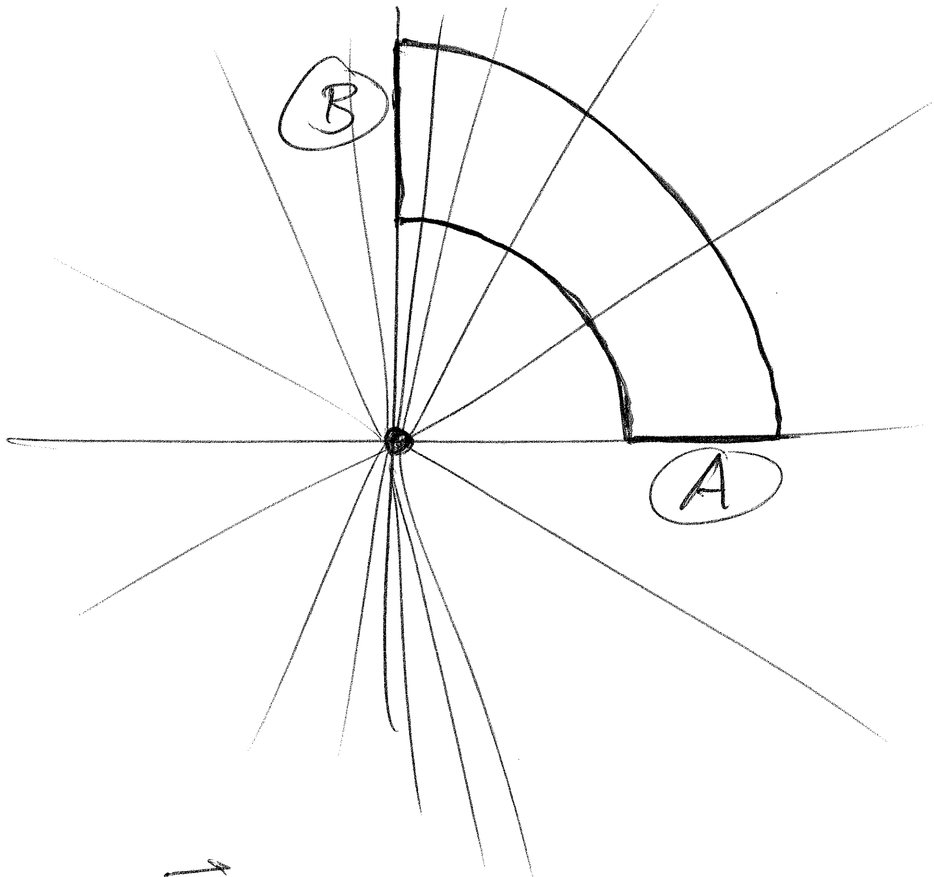


As with a charge at rest, the field is radial and may be represented by field lines that never start or stop (except at the charge) since Gauss's law continues to be valid. The only thing that's changed is a bunching of the field lines in the plane perpendicular to the motion and a thinning along the axis of motion. The field-line snapshot does not tell us if the charge is moving right or left.

Q: Does the field at a particular instant correspond to a <u>conservative force field</u>?

It's easy to see why the answer to the question is "no".

Consider moving a test charge around a loop that comprises two quarter-arcs at different radius connected at $\theta = 0$ and $\theta = \pi/2$:



Because $\vec{E}$ is radial, work is performed only at the radial segments A and B. But the magnitudes of $\vec{E}$ at A and

B differ by the angular factor

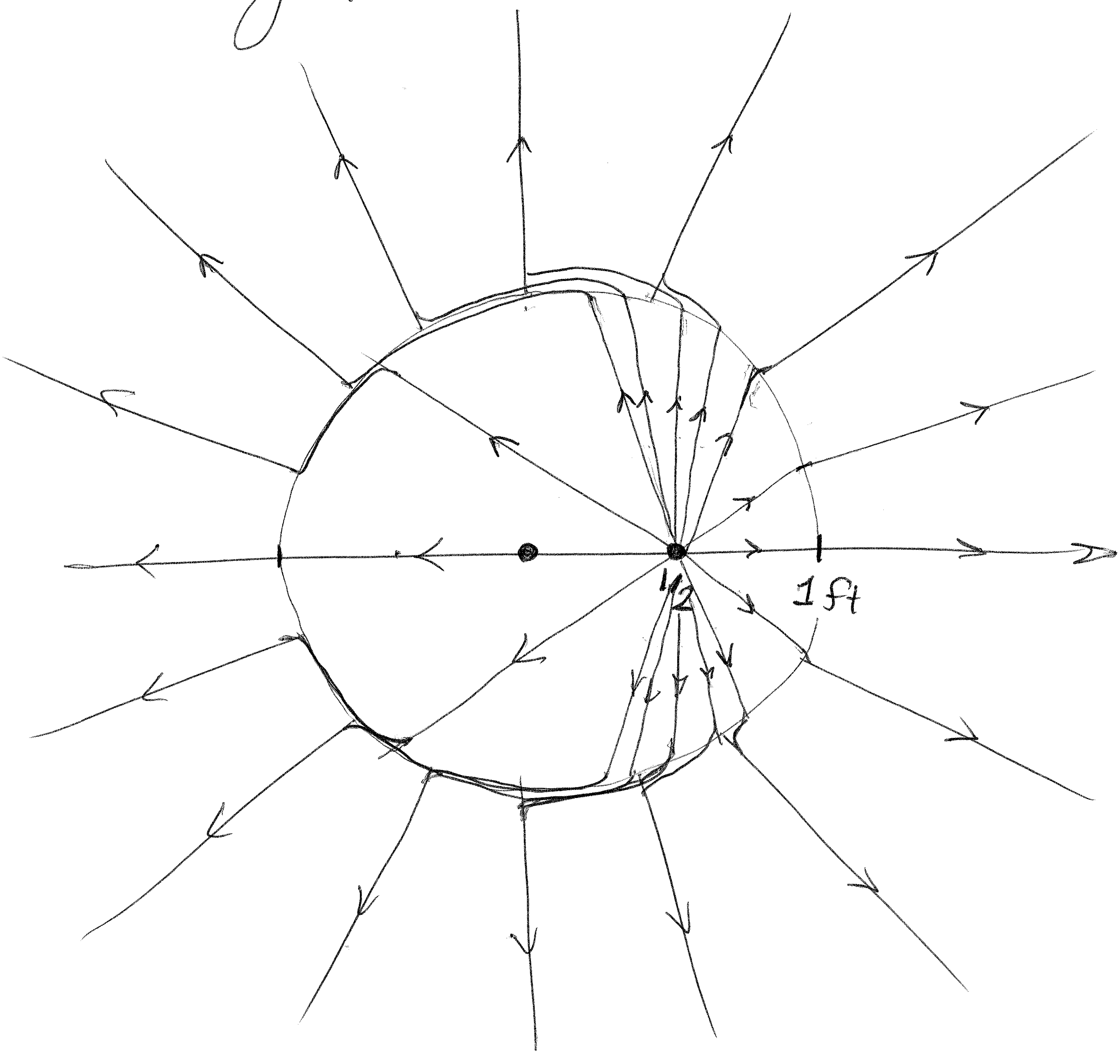
$$\frac{1-\beta^2}{(1-\beta^2\sin^2\theta)^{3/2}} = \begin{cases} 1-\beta & \theta = 0 \quad \text{A} \\ \dfrac{1}{\sqrt{1-\beta^2}} & \theta = \pi/2 \quad \text{B} \end{cases}$$

This does not create energy-conservation problems because it takes a finite time to move a test charge around the loop and in that time the field will have changed (because the source/point-charge is moving).

The derivation we used to find the field of a uniformly moving charge relied heavily on special relativity (inertial frames) and cannot be extended to deal with accelerating charges. However, there is an idealized limit of accelerated motion that we can analyze. This is when the motion switches abruptly between different states of uniform motion. As an example, let's suppose the charge is at rest at the origin until $t=0$, whereafter it begins moving to the right with speed $v=c/2$. We will try to reconstruct the electric field <u>at time $t=1\,ns$</u>

(10^{-9} sec). Our x-axis will be marked-off in units of feet since $c \simeq 1$ foot/ns.

Now at time $t = 1$ ns all points 1 foot and greater from the origin will still "think" the charge is at rest since the information about the start of motion cannot propagate faster than c. There will thus be a "bubble" of radius 1 foot outside of which we have the spherically symmetric field of a charge at rest. Inside the bubble the field will be that of a moving charge which will have arrived at $x = 1/2$ foot at $t = 1ns$ since $v = 1/2\,c$. Stitching the two field patterns together we get the following picture:



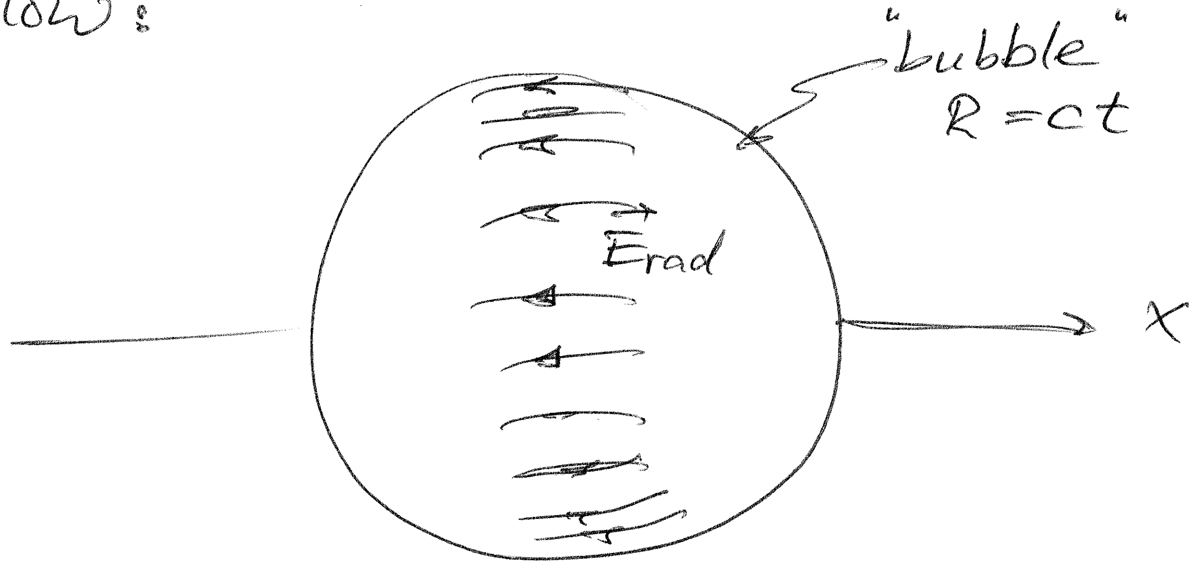
Note that we have connected up field lines inside and outside the bubble to be consistent with Gauss's law. To do that we had to ~~insert~~ thread field lines within the thin bubble surface (the thickness of which is determined by a finite acceleration time). The existence of such surface field lines is the most interesting thing to have emerged from this exercise. We will interpret them as the electric field associated with radiation — in fact, the ~~a~~ burst of radiation created by sudden acceleration. Even though our constuction of this radiation field lacks mathematical rigor, it correctly captures several qualitative properties of radiation fields:

- light-speed propagation (bubble radius $R(t) = ct$)

- $\vec{E}_{rad}$ <u>perpendicular</u> to propagation direction

- $|\vec{E}_{rad}| \propto 1/R$

The last property, decay as the inverse first power of the distance to the source, is in contrast to non-radiation fields that decay at least as $1/R^2$. The $1/R$ property comes from the fact that there is a <u>fixed number</u> of field lines around the azimuth of the bubble as shown for θ near $\pi/2$ below:



Since the bubble thickness is constant (set by finite acceleration time), the strength of $\vec{E}_{rad}$ is proportional to the density of lines within the bubble surface:

$$|\vec{E}_{rad}| \propto \frac{\#\text{ lines}}{2\pi R} \propto 1/R .$$