

## Lecture 21

(1)

An observer moving in a direction parallel to the charged sheets will see a charge density  $\sigma' = \gamma\sigma$  that is enhanced relative to the charge density  $\sigma$  in the rest frame of the sheet. And since the symmetry of  $\vec{E}'$  is the same in the moving frame, by Gauss's law the magnitude of  $\vec{E}'$  will be enhanced by the same factor  $\gamma$ :

$$|\vec{E}'| = \gamma |\vec{E}| \quad \left( \begin{array}{l} \text{motion parallel} \\ \text{to sheets} \end{array} \right)$$

Let's now repeat the previous exercise for an observer moving perpendicular to the charged

② sheets. Again, it's easy to see that this cannot change the symmetry of  $\vec{E}'$ . But in this case not even the magnitude of  $\vec{E}'$  will change because both dimensions of the charge density are perpendicular to the direction of motion and therefore do not experience "Lorentz contraction":

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$$|\vec{E}'| = |\vec{E}| \quad (\text{motion perp. to sheets})$$

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We are now ready to make a more general statement, one that does not refer specifically to infinite charged sheets. Suppose an observer

in some inertial frame sees (3)  
an electric field  $\vec{E}$ . On a sufficiently  
small scale this field can be  
approximated as uniform. How  
will this field appear to another  
observer in relative motion with  
respect to the first observer?

We already know the answer to  
this question when the field  $\vec{E}$   
happens to be produced by sheets  
of charge and the motion is  
either perpendicular or parallel  
to  $\vec{E}$  :

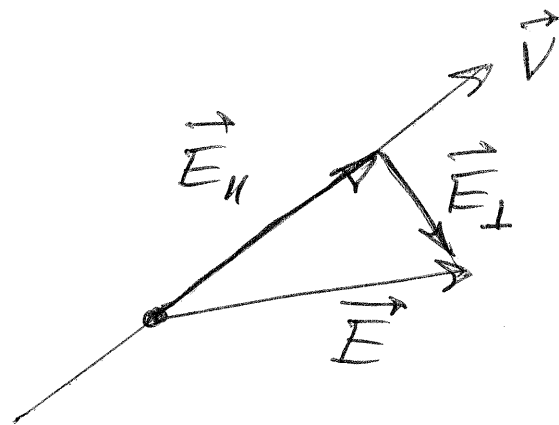
$$\left( \begin{array}{l} \text{motion } \perp \\ \text{to } \vec{E} \end{array} \right) : \vec{E}' = \gamma \vec{E}$$

$$\left( \begin{array}{l} \text{motion } \parallel \\ \text{to } \vec{E} \end{array} \right) : \vec{E}' = \vec{E}$$

But if the electric field is a true physical entity ("the thing that causes electric force on charged particles") then the specific mechanism that created it should not matter. Also, once the direction of motion is specified, we can uniquely decompose  $\vec{E}$  and  $\vec{E}'$  into parts that are parallel and perpendicular to the motion:

$$\vec{E} = \vec{E}_{\parallel} + \vec{E}_{\perp}$$

$$\vec{E}' = \vec{E}'_{\parallel} + \vec{E}'_{\perp}$$



The transformation rule we found can be neatly summarized as follows:

(5)

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel}$$

$$\vec{E}'_{\perp} = \gamma \vec{E}_{\perp}$$

Our next task will be to use this simple rule to work out the electric field of the simplest moving source: the point charge.

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