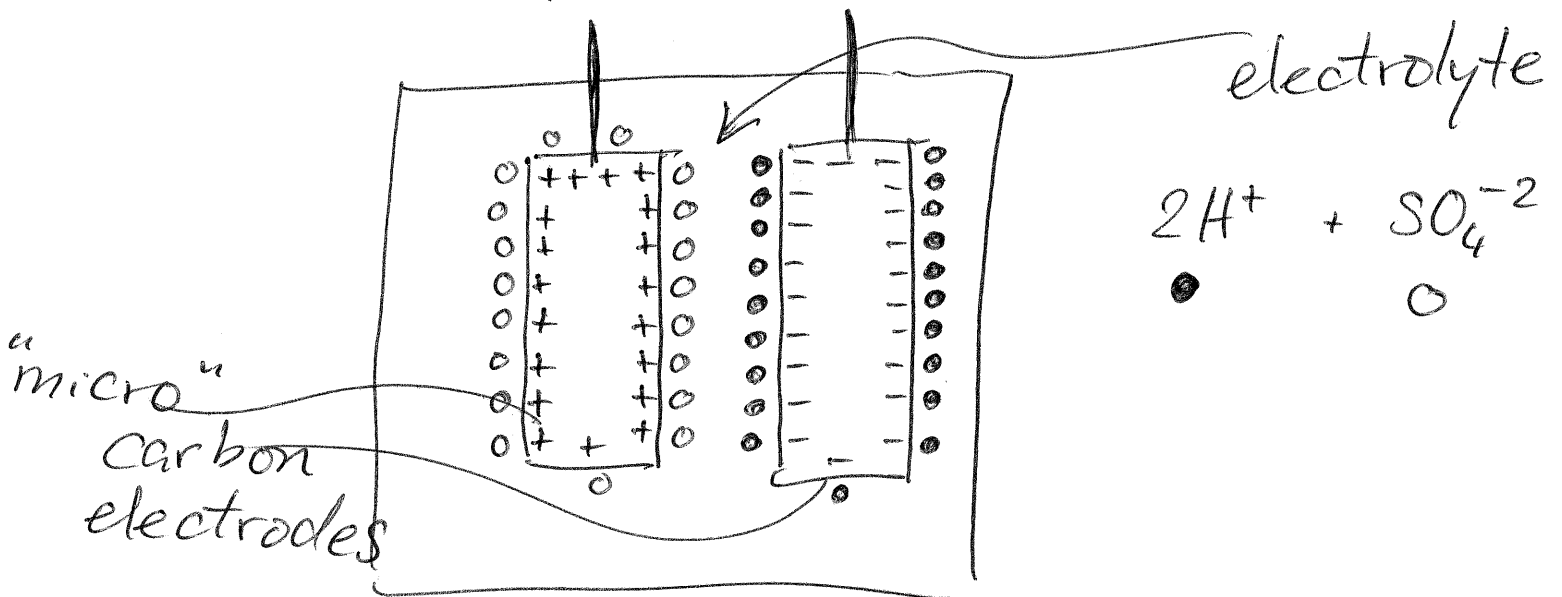# Lecture 20
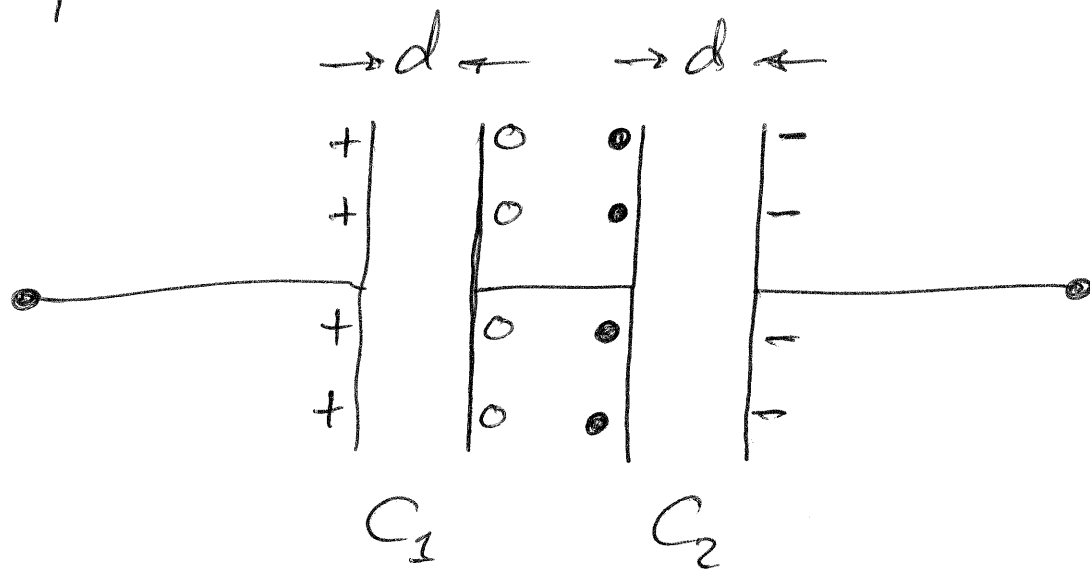
Conceptual physics exercise :
series vs. parallel lumped capacitors.
Of Q and V, which are shared and
which are summed ?

Here is a schematic drawing
showing the construction of the
1F "super capacitor" :

electrolyte

$2H^+ + SO_4^{-2}$

"micro"
carbon
electrodes

This is actually like two
capacitors in series:

$$C_{series} = \left(\frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}$$

The ions coat the surface of the
carbon electrodes so the separation
of the "plates" is only on the order
of a molecular diameter!

$$d \simeq 10\text{Å} = 10^{-9}\,m = 1\,nm$$

We will model C_1 and C_2 as
parallel plate capacitors of area A.

$$C_1 = C_2 = \epsilon_0 \frac{A}{d}, \quad C_{series} = \frac{C_1}{2}$$

$$\frac{C_1}{A} = \frac{\epsilon_0}{d} \sim \frac{10^{-11} \, F/m}{10^{-9} \, m} = 10^{-2} \, F/m^2$$

$$C_{series} = 1 \, F \Rightarrow C_1 = 2F$$

$$\Rightarrow A = 200 \, m^2 \, !$$

It's possible to fit $200 \, m^2$ area into a space that you can hold in your hand by compacting into a solid <u>small particles</u> of carbon:

small carbon particles with enormous interstitial area

possible to get $\dfrac{1000 \, m^2}{gm}$

$$\Rightarrow \tfrac{1}{5} gm \; carbon/electrode$$

In the next few lectures we will
be comparing the electric field, at
a particular space-time event, as
observed ~~by~~ in different inertial
reference ~~fa~~rames. As a 3-vector,
we already know o how $\vec{E}$ and $\vec{E}'$
are related when one frame is
simply rotated with respect to the
other. What about frames in
relative (uniform) motion? If $\vec{E}$
could be generalized as a 4-vector,
then we would know what to do.
However, that turns out <u>not</u> to be
the case. The transformation rule,
relating $\vec{E}$ and $\vec{E}'$ in relatively moving
frames, is actually quite simple

and we will ~~the~~ arrive at it by   ⑤
applying very basic principles. These
include :

- symmetry
- special theory of relativity
  (as it relates distances)
- invariance of charge
- field principle
  ($\vec{E}$ exists as a local entity,
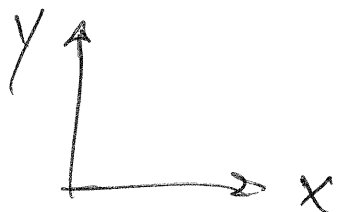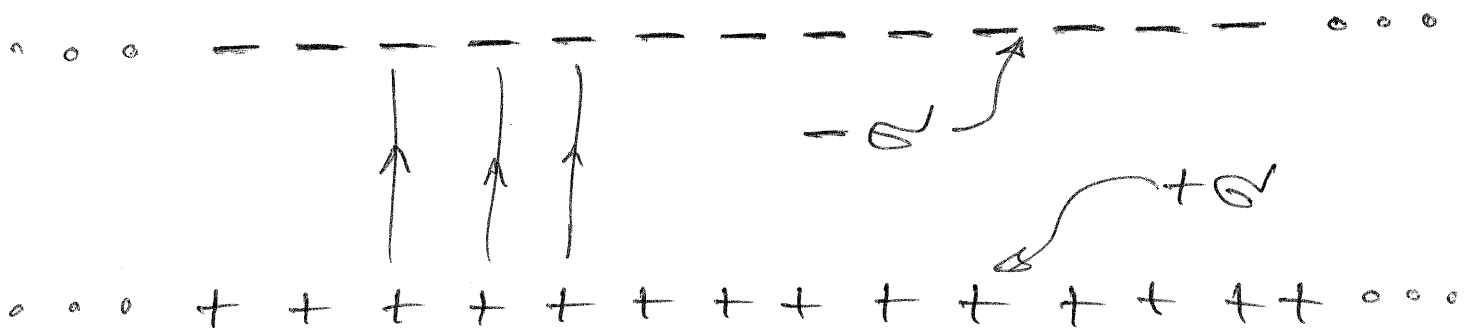   independent of its sources.)

We'll start by comparing the electric
fields produced by ~~the~~ very symmetric
distributions of charge — infinite
planar sheets — when the motion
is ~~the~~ ~~the~~ parallel to the planes. A
uniform sheet of charge will still se

a uniform sheet of charge in
the moving frame. One thing that <u>will</u>
be different is the <u>density</u> of charge
— a consequence of "Lorentz contraction".

However, according to the "invariance
of charge", the quantity of charge
enclosed by a Gaussian surface is
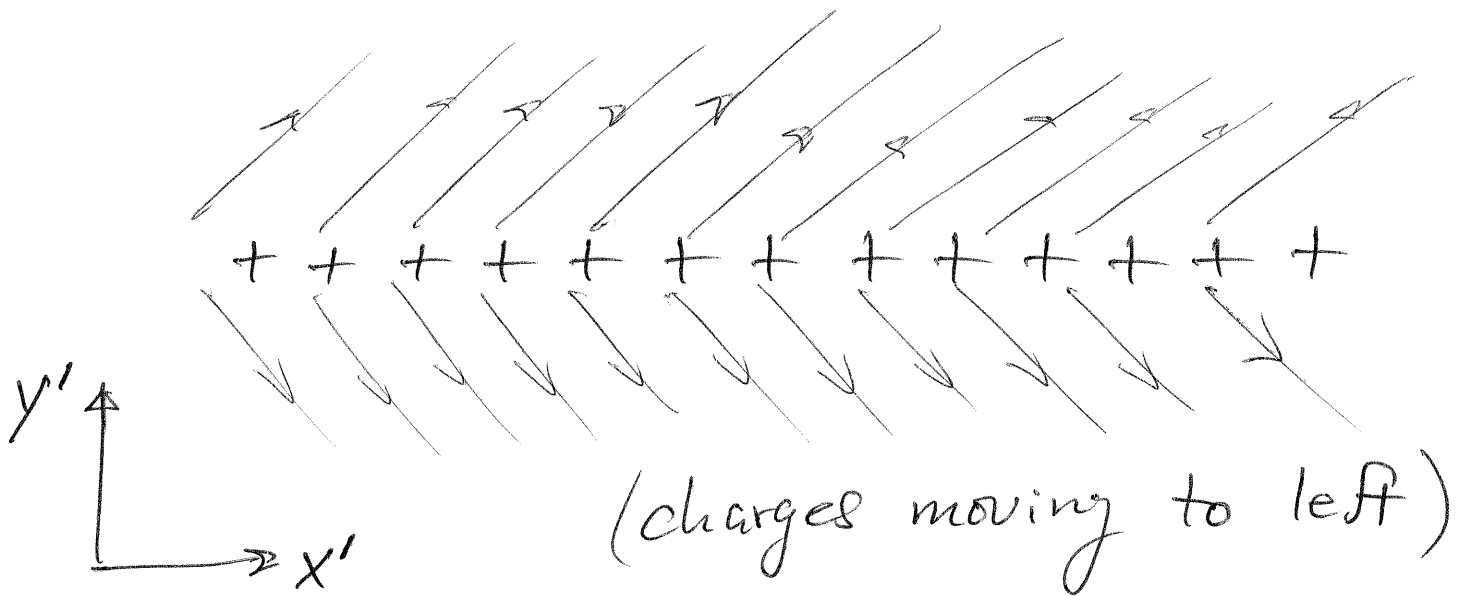unchanged by the state of motion of
the charge.

Our sheets of charge will be
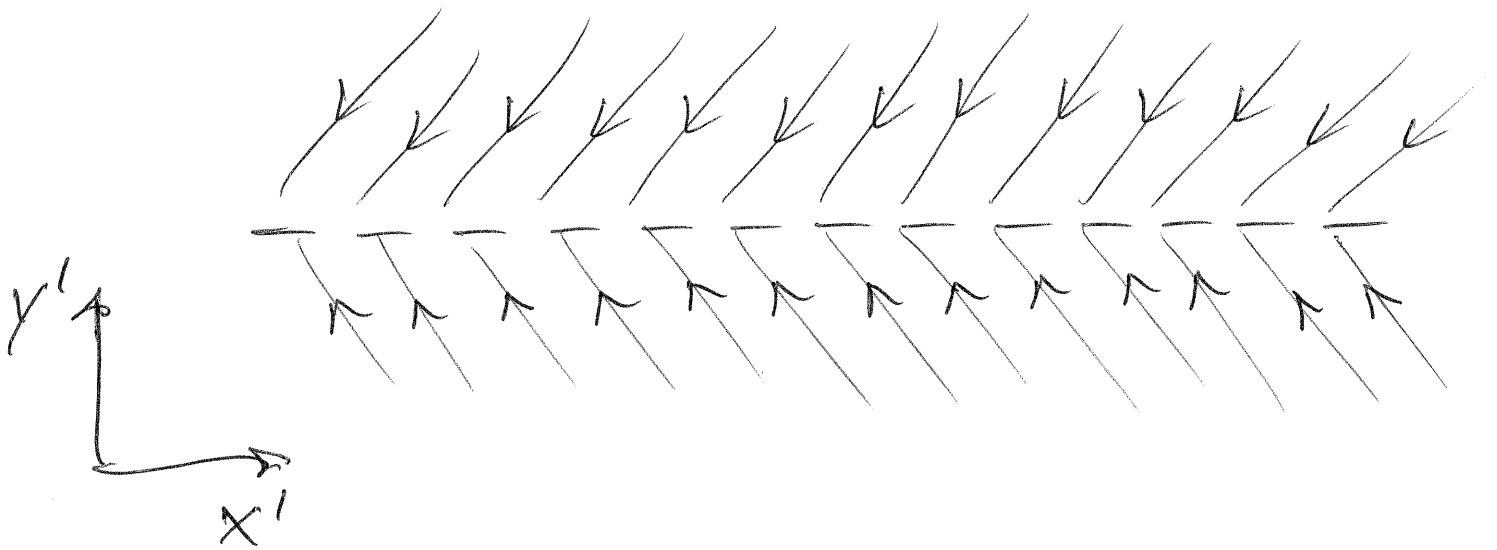rendered in cross section like this:

We have two parallel sheets
in the x-z plane with charge
density $\pm\sigma$ in the rest frame of
the charge. The electric field will
be purely in the y-direction with
magnitude σ/ε_0.

What electric field is seen by
an observer ~~moving~~ moving with speed
v in the x-direction? The
vanishing of $\vec{E}$ above and below the
sheets and its perpendicular direc-
tion (relative to the sheets) will
be unchanged by the motion of
the charges in this frame. It's
true that the state of motion of
the charges undermines the right-
left symmetry of the x-axis, so

the field produced by just the
+σ sheet might look like :

(charges moving to left)

The field of the −σ sheet would
then be

Superposing these gives a
perpendicular field that is nonzero

only between the sheets—exactly as in the rest frame. So the only thing that might be different is the <u>magnitude</u> of $\vec{E}'$. We'll determine this using Gauss's law, which we know (from experiments with rapidly moving elementary charges) is valid even when the charges enclosed are moving. Now a Gaussian surface at rest in the primed (moving) frame will enclose an amount of charge that we can calculate using the rules of special relativity. The spacing of charges will be contracted

in the x' direction and unchanged in the z' direction, perpendicular to the motion. The contraction in x' is by the factor
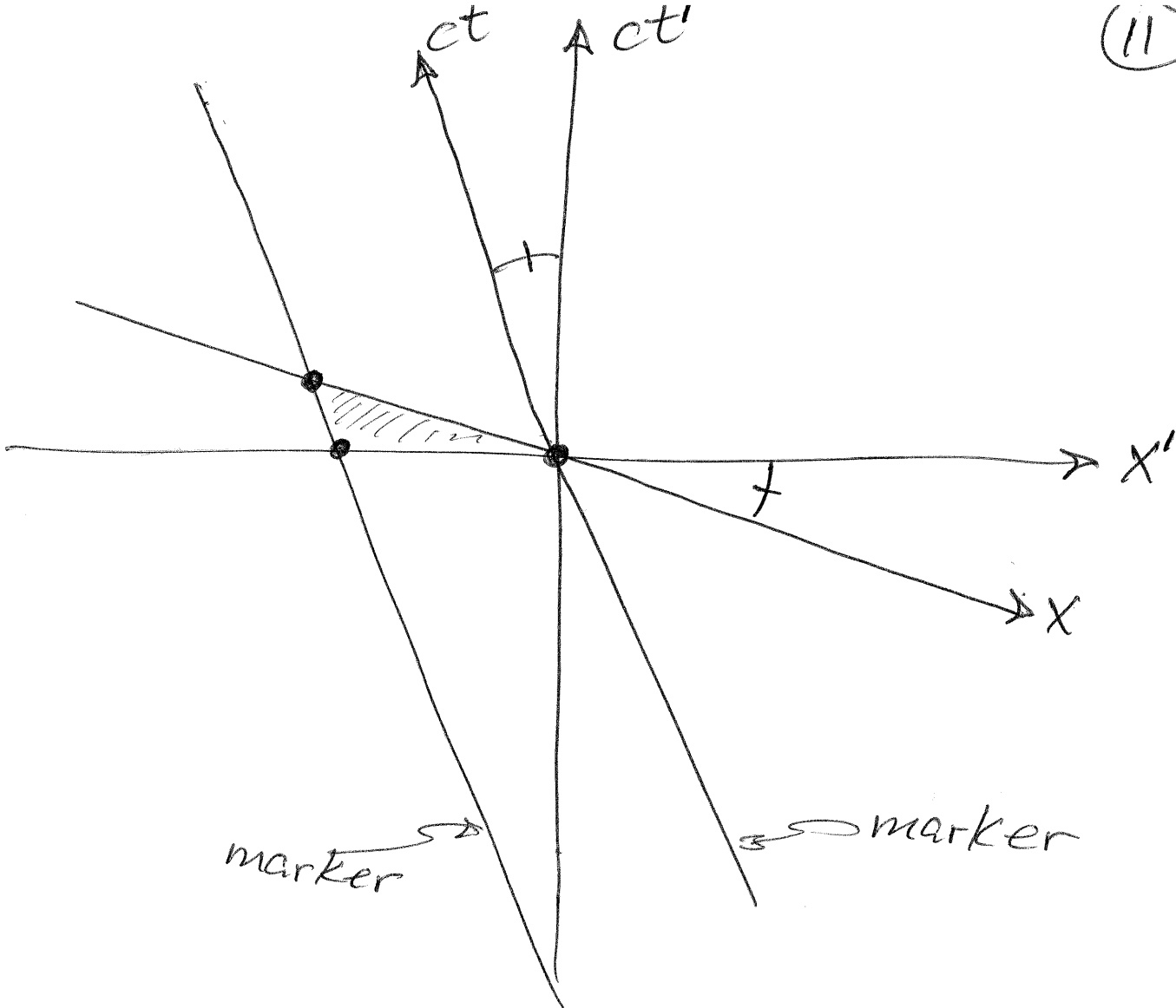
$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \geq 1,$$

and so the charge density in the moving frame is enhanced as

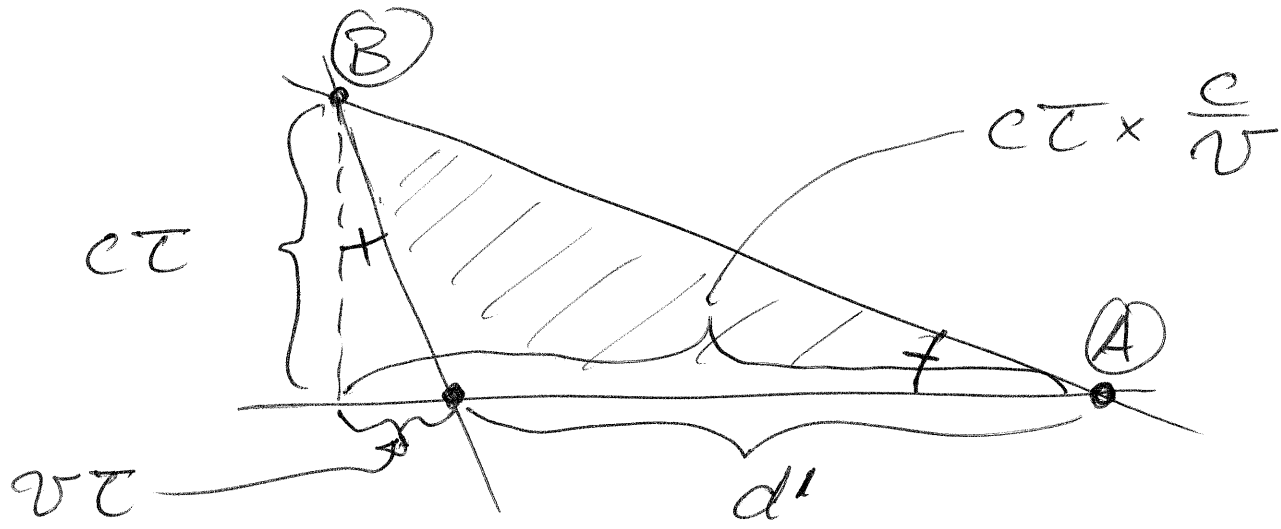$$\sigma' = \gamma \sigma.$$

Here is a quick review of the Lorentz contraction effect. We will focus on two fixed markers in one sheet at different x in the rest frame; these will have slanted world lines in the moving frame:

Now we do some similar triangle geometry on the triangle highlighted above:

$$d' = \begin{pmatrix} \text{distance between markers} \quad \text{(12)} \\ \text{in primed frame, when} \\ \text{they are simultaneous in that} \\ \text{frame} \end{pmatrix}$$

$$d' = \frac{c^2}{v}\tau - v\tau = \frac{c^2}{v}\tau\left(1 - \frac{v^2}{c^2}\right)$$

$$d = \begin{pmatrix} \text{distance between markers} \\ \text{in un-primed frame} \end{pmatrix}$$

$$= \begin{pmatrix} \text{proper distance between} \\ \text{events } \text{\textcircled{A}} \text{ and } \text{\textcircled{B}} \end{pmatrix}$$

$$= \sqrt{\left(\frac{c^2}{v}\tau\right)^2 - (c\tau)^2}$$

$$= \sqrt{\left(\frac{c}{v}\right)^2 - 1} \; c\tau$$

$$= \sqrt{1 - \frac{v^2}{c^2}} \; \frac{c^2}{v}\tau$$

$$\frac{d'}{d} = \frac{1 - v^2/c^2}{\sqrt{1 - v^2/c^2}} = \sqrt{1 - v^2/c^2} = \frac{1}{\gamma}$$

(13)