

Lecture 1

(1)

The first thing usually discussed in a course on electricity and magnetism is Coulomb's law.

Coming on the heels of a mechanics course this makes a lot of sense, electric force being one of the most important forces in everyday phenomena.

However, we will see that Coulomb's law is not "just another force" that mechanics has taught us to deal with, but is an entry point for a type of physics that is bigger than what is usually seen as the domain of mechanics.

Mathematically, Coulomb's law ⁽²⁾ is an expression for the force acting between a pair of point charges q_1 and q_2 located at positions \vec{r}_1 and \vec{r}_2 in empty space:

$$\vec{F}_{21} = \text{force on charge 2 due to charge 1}$$

$$= k \frac{q_1 q_2}{r_{12}^2} \hat{r}_{21}$$

$$r_{12} = r_{21} = |\vec{r}_1 - \vec{r}_2|$$

$$\begin{aligned} \hat{r}_{21} &= \text{unit vector from 1 to 2} \\ &= -\hat{r}_{12} \end{aligned}$$

$K =$ universal physical constant

(3)

$$\approx 9 \times 10^9 \text{ N } \frac{\text{m}^2}{\text{C}^2}$$

C = coulomb = MKS unit of charge

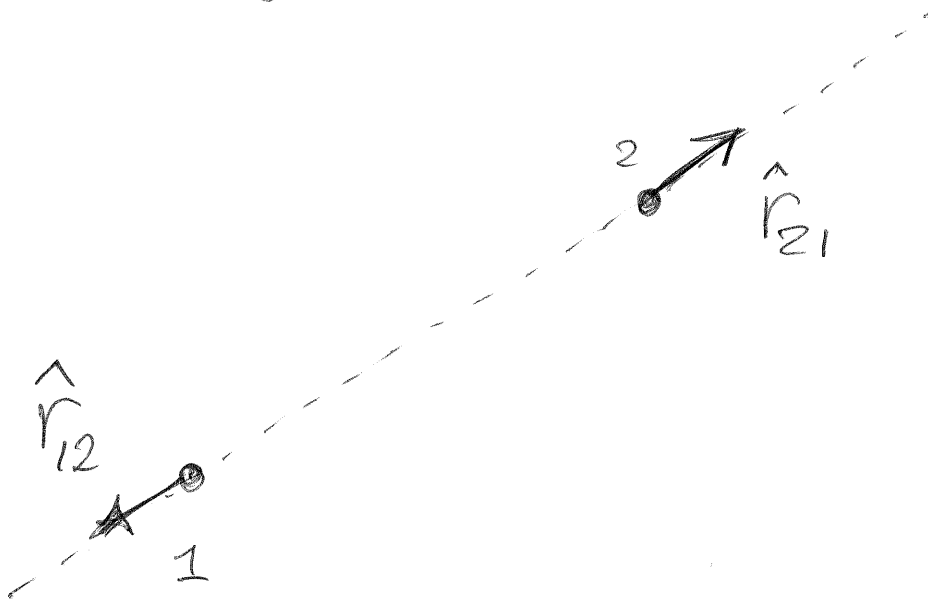
Q: what is the electric force between a proton and electron in a hydrogen atom?

$$|q_1| = |q_2| = e \approx 1.6 \times 10^{-19} \text{ C}$$

$$r_{12} \approx 1 \text{ \AA} = 10^{-10} \text{ m}$$

$$F \approx 10^{10} \left(\frac{1.6 \times 10^{-19}}{10^{-10}} \right)^2 \approx 3 \times 10^{-8} \text{ N}$$

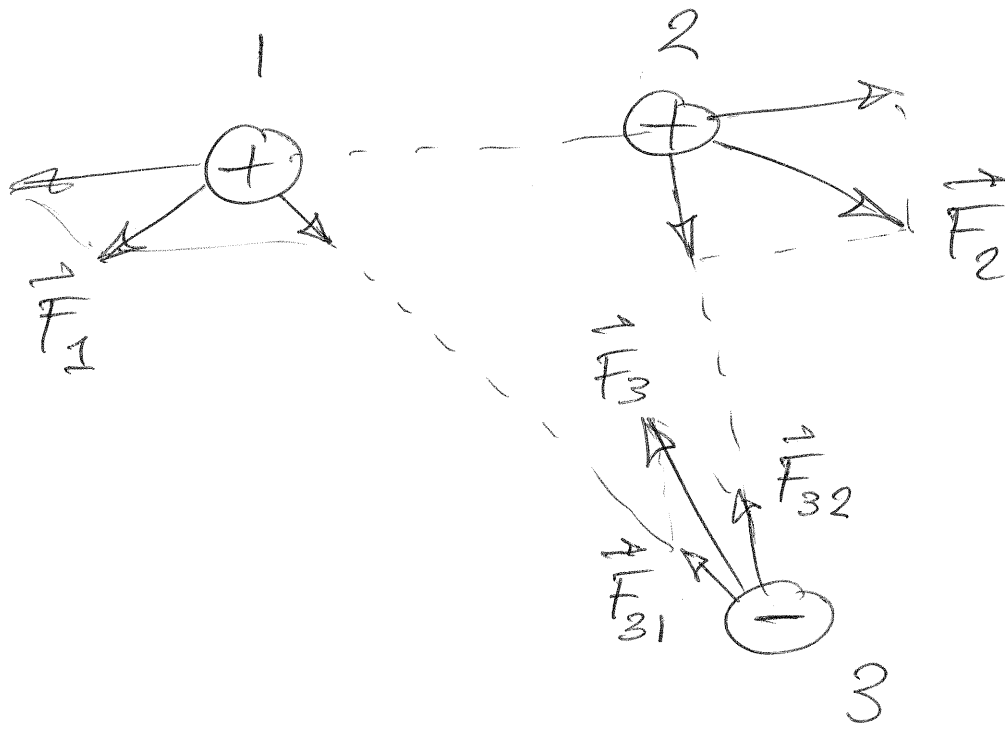
The vectorial structure of (4) Coulomb's law is consistent with the symmetry of space and the scalar (non-vector) nature of charge. If space is truly isotropic, then the direction of the force can only be along the line that runs through the two charges (\hat{r}_{21}):



Since the charge scalars appear symmetrically, as the product $q_1 q_2$, Newton's 3rd law $\vec{F}_{12} = -\vec{F}_{21}$ follows directly from the vector identity $\hat{r}_{12} = -\hat{r}_{21}$. The "non-trivial" content of Coulomb's law therefore reduces to the following:

- (1) inverse-square behavior
- (2) possibility of attractive and repulsive force given both signs of charge
- (3) magnitude of force given ~~standard~~ observed elementary charges (protons, electrons)!

There is additional content (6) in Coulomb's law coming from the not-yet-articulated rule for how it is to be applied when there are more than two charges. The name for this rule is superposition. We say all the pair-wise Coulomb forces acting on a particular charge, from a collection of other charges, are added-up (as vectors) or "superposed" to give the actual force. Here is an example with three charges :



The superposition rule will seem more natural when we think of the Coulomb force as actually being caused by the electric field, and the charges as sources of the field.