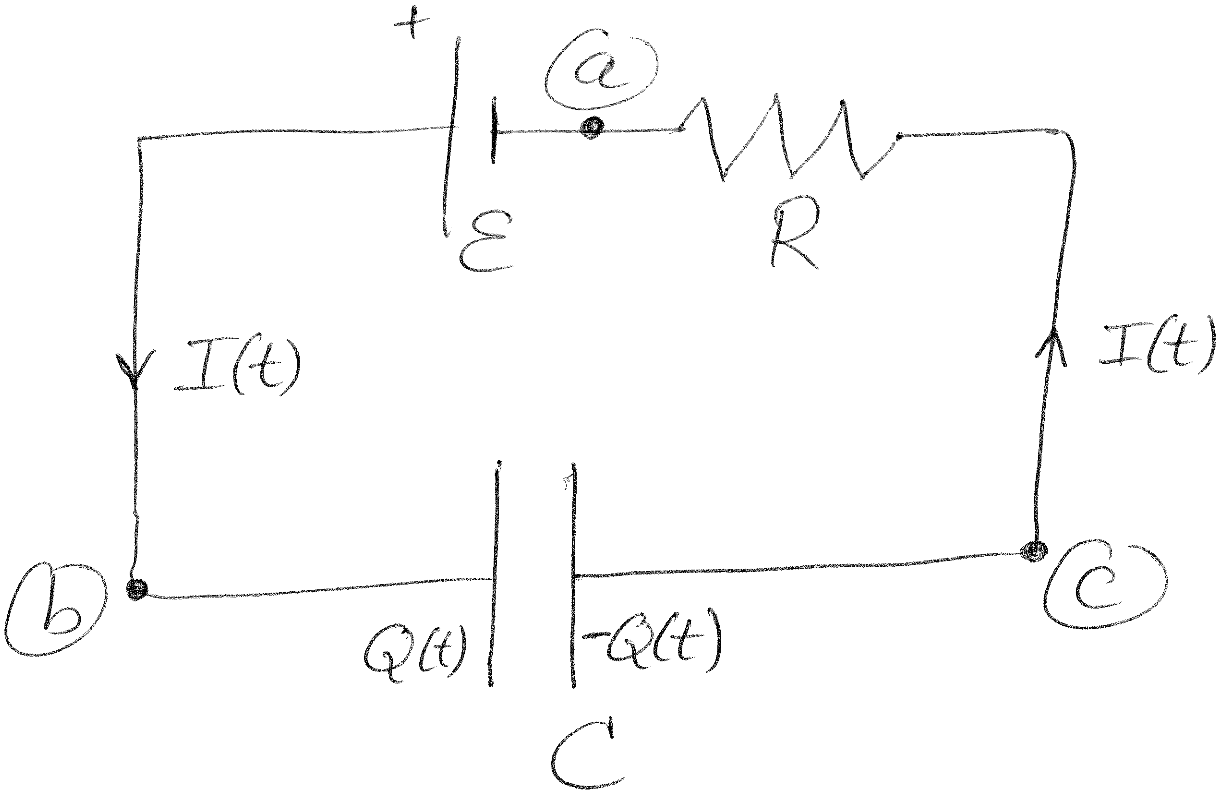
Lecture 19

Let's apply the "loop rule" to the capacitor charging circuit:



The pure emf raises the potential by $+\mathcal{E}$ as we go from ⓐ to ⓑ. The voltage across the capacitor is $V = Q(t)/C$ (definition of ②

the capacitance C) where higher potential terminal is the one with positive charge (since $\vec{E}$ is directed from the positive plate to the negative plate). Therefore, if $Q(t) > 0$ in the diagram, the potential will drop by $-Q(t)/C$ as we go from ⓑ to ⓒ. This change in potential is also correct if $Q(t) < 0$; in that case the potential increases (the direction of $\vec{E}$ is reversed) and $-Q(t)/C$ is a potential rise (positive quantity).

Finally, in going from c back to a through the resistor the potential drops by $-IR$ (if it turned out that I was negative the potential in the resistor would drop when moving in the opposite direction — it always drops when moving in the direction of positive current). The three potential changes must bring us back to the original potential:

loop rule: $+\mathcal{E} - \frac{Q(t)}{C} - I(t)R = 0$

This equation contains two unknowns: $Q(t)$ and $I(t)$. We need another equation that relates these unknowns. What's needed is the statement of charge conservation applied to the individual plates of the capacitor:

$$\frac{dQ(t)}{dt} = I(t) \qquad \text{(left plate)}$$

If this is satisfied then

$$\frac{d(-Q(t))}{dt} = -I(t) \qquad \text{(right plate)}$$

is also true and we have charge conservation ~~#~~ on both plates.

Substituting $\dot{Q}$ for I in the loop equation we get the following differential equation:

$$\mathcal{E} - Q/C - \dot{Q}R = 0$$

The corresponding homogeneous equation ($\mathcal{E} \to 0$) is

$$\dot{Q}_h = -\frac{1}{RC} Q_h$$

with general solution

$$Q_h(t) = Q_h(0)\, e^{-t/RC} .$$

A <u>particular</u> solution of the

loop equation is

$$Q(t) = C\mathcal{E} \quad (= \text{const.})$$

The most general solution is then this solution plus the most general solution of the homogeneous equation:

$$Q(t) = C\mathcal{E} + Q_h(0)\, e^{-t/RC} .$$

Different choices of the constant $Q_h(0)$ corresponds to different scenarios or <u>initial conditions</u>.

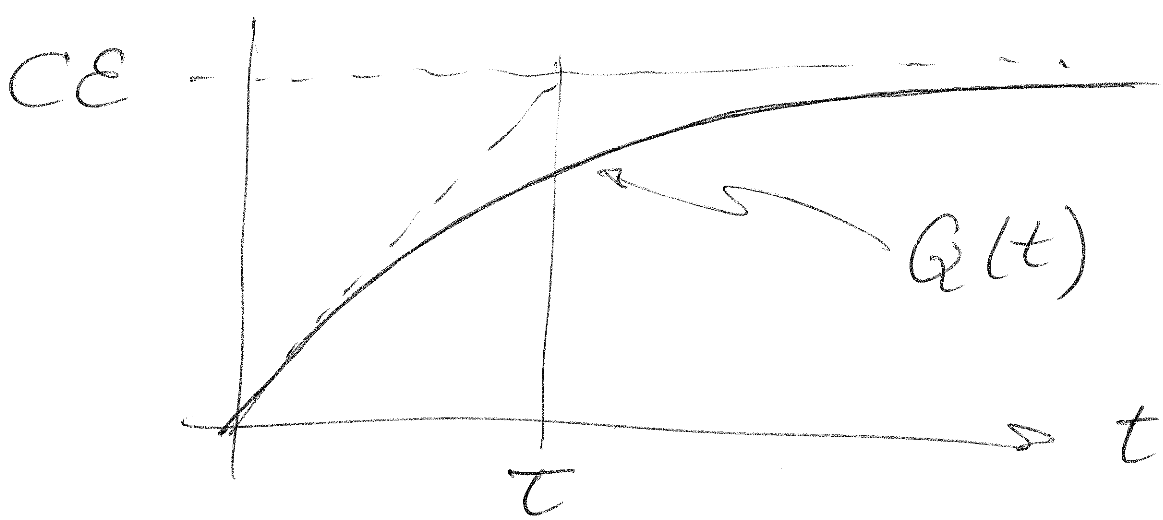
If initially (t=0) the capacitor is uncharged, then we want $Q_h(0) = -C\mathcal{E}$:

charging solution :

$$Q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)$$

$\tau = RC$ = "time constant"

Ohm × Farad = second



The charge approaches the static equilibrium ($I=0$) value

$Q = C\mathcal{E}$ on an exponentially diminishing asymptote. A straight-line extrapolation of the initial slope,

$$I(0) = \dot{Q}(0) = \frac{C\mathcal{E}}{\tau}$$

would reach $Q = C\mathcal{E}$ in the time τ.

A "charged" or energized capacitor can be directly connected to a circuit without a battery and will drive a current. Our circuit diagram for charging

still applies if we set $\mathcal{E}=0$ (no battery) and interpret the resistance as the device (light-bulb, etc.) we are driving.
The equation is then

$$\dot{Q} = -\frac{1}{\tau} Q \quad ,$$

with general solution:

$$Q(t) = Q(0)\, e^{-t/\tau} \qquad \text{"discharging"}$$