Lecture 18

A capacitor is defined most generally as a device that can store charge. The charge is stored on a conductor ~~that~~ whose potential — relative to infinity or some other reference — is proportional to the charge stored. We have already seen examples of this. Take a conducting sphere of radius R and place charge Q on it. In equilibrium the charge will be uniformly distributed over the surface, and the field $\vec{E}$ and potential φ for $r > R$ will be identical with that of a point charge Q located at the sphere center. The potential of the conductor is therefore

$$V = \varphi(R) = K\frac{Q}{R} .$$

The potential is proportional to the charge and their ratio is the "capacitance" C:

$$\underbrace{C \equiv \frac{Q}{V}}_{\text{definition}} = \underbrace{\frac{R}{K} = 4\pi\epsilon_0 R}_{\text{spherical geom.}}$$

The units of C are

$$\frac{\text{Coul}}{\text{Volt}} \equiv \text{Farad } (F).$$

From the last equality we see that capacitance has units of $\epsilon_0 \times$ length. We can turn this around and obtain alternative units for ϵ_0:

$$\epsilon_0 \cong 8.85 \, pF/m$$

$$= 8.85 \times 10^{-12} \, F/m$$

So a 1 meter radius sphere would store $4\pi \times 8.85 \times 10^{-12}$ Coulombs of charge when its potential is 1 Volt. Given the smallness of ϵ_0 in these units it would appear that a 1F capacitor would be impractically huge. However, in the next lecture we will describe the construction of 1F capacitors that fit in the palm of your hand.

In actual circuits we want our component devices to be relatively isolated — an electric field that impinges on the rest of the circuit would not be a practical design. Most capacitors therefore consist of two conductors that hold equal and opposite charge, Q and $-Q$, such that the electric field is confined to a small space. The canonical geometry is the parallel-plate capacitor:

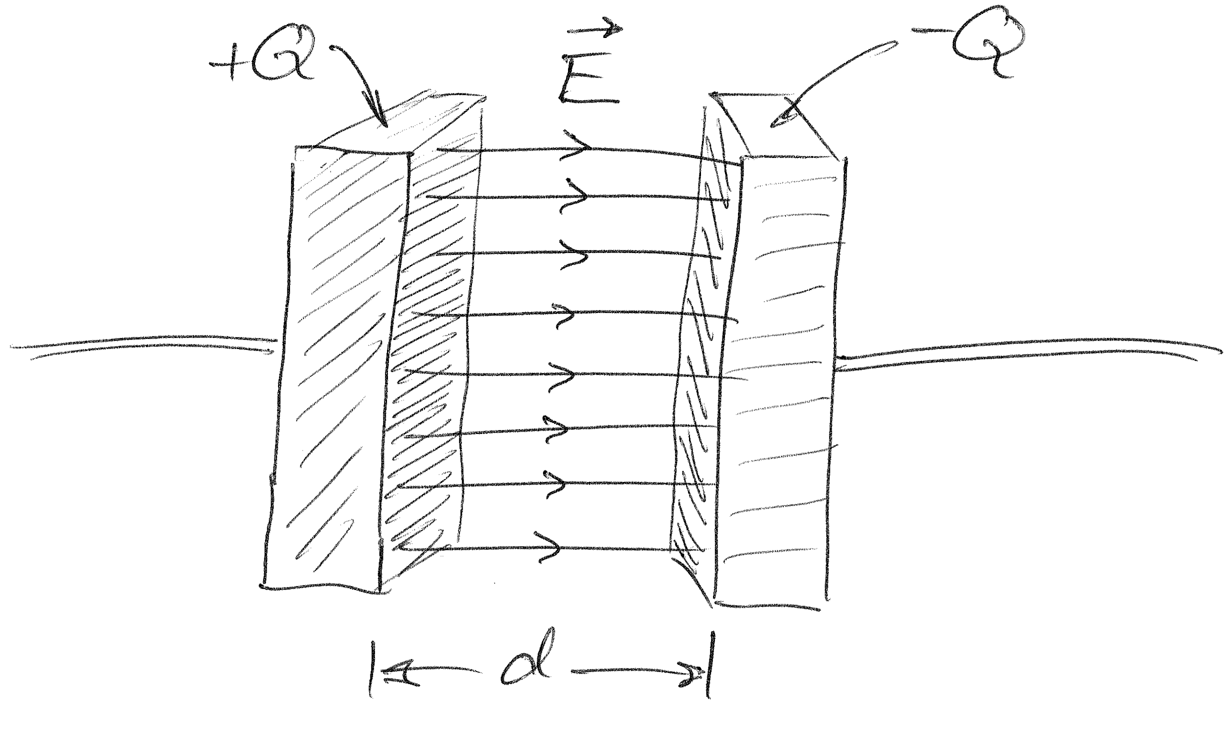


plate area = A (each side)

We'll assume the plates are very large on the scale of the spacing d ($A \gg d^2$) so the electric field is very uniform over most of the space between the plates. The surfaces of the plates (facing the interior) will then have a very uniform surface charge density (lecture 9):

$$\sigma = \epsilon_0 E$$

(of opposite sign on the two plates). The charge stored on one plate is therefore

$$Q = \sigma \cdot A = \epsilon_0 A E$$

writing the magnitude of the uniform electric field E in terms of the potential difference between the plates,

$$E = V/d \,,$$

we obtain the proportionality between Q and V as before:

$$Q = \epsilon_0 A (V/d)$$

$$\Rightarrow C = \epsilon_0 \frac{A}{d}$$

The greater the stored charge Q, the greater will be the electric field and consequently, the stored <u>energy</u>. For the parallel plate geometry the energy is easy to compute because the field and energy density is uniform:

$$U = u \times (\text{volume occupied by electric field})$$

$$u = (\text{electric energy density}) = \frac{\epsilon_0}{2} E^2$$

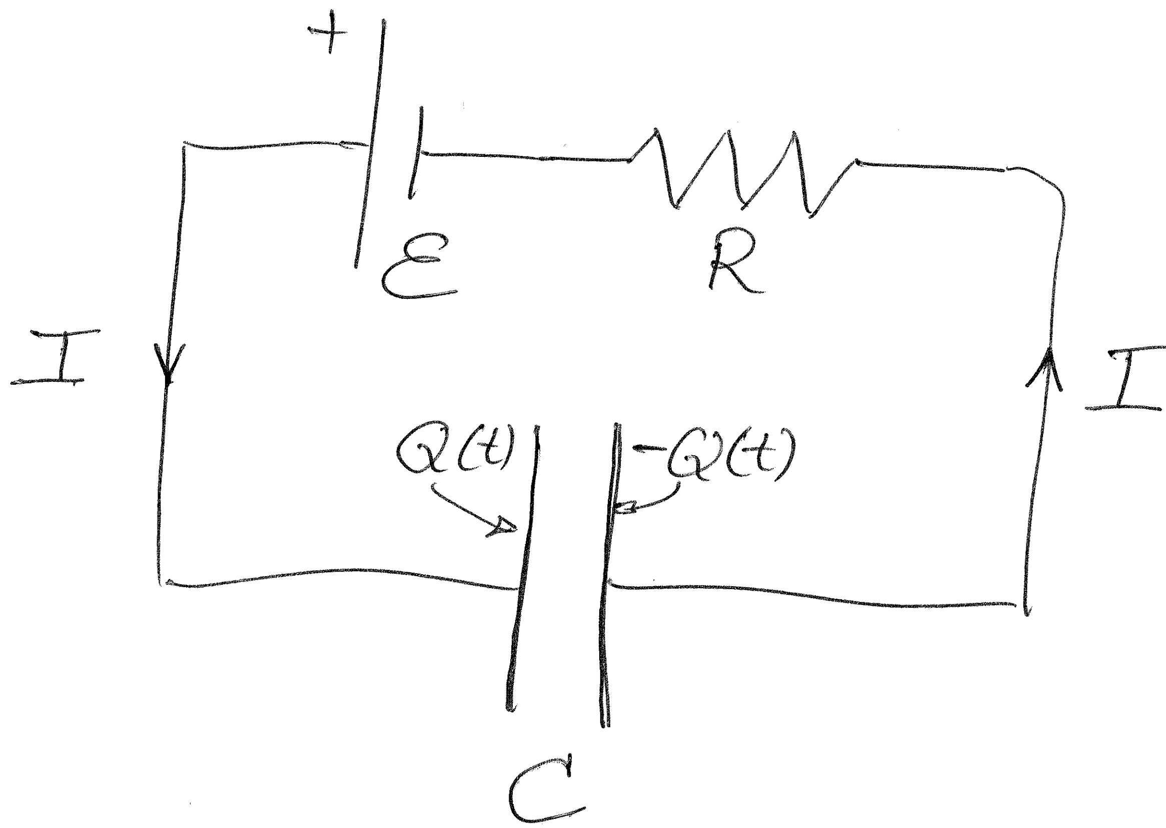
$$\Rightarrow U = \frac{\epsilon_0}{2} E^2 A d$$

$$= \frac{\epsilon_0}{2} \underbrace{(E \cdot d)}_{V}^2 \frac{A}{d} = \frac{1}{2} C V^2$$

Using the definition $C = Q/V$ we can write this also in terms of Q:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\,Q^2/C \,.$$

If we connect the terminals of an uncharged capacitor to the terminals of a battery, there will be a flow of charge from the battery to the capacitor plates until the potential across the capacitor equals the battery emf. The energy stored in the capacitor's electric field is provided by the chemical energy in the battery. We will analyze this "charging" process by deriving and solving equations for the corresponding circuit diagram:



We have seen the pure emf and resistor before (the latter would be the internal battery resistance). The new circuit icon, a pair of parallel lines (plates), represents the capacitor. Because there is no net charge flowing into the battery (note current arrows), there is also no <u>net</u> charge flowing into the capacitor. However, there <u>is</u> net charge $\pm Q(t)$ accumulating on the two capacitor plates. Working out the function $Q(t)$ will be our next task.