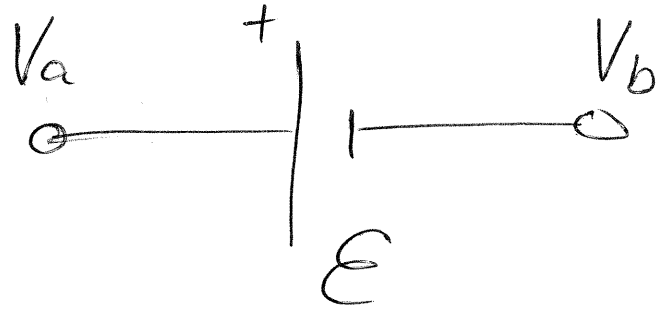
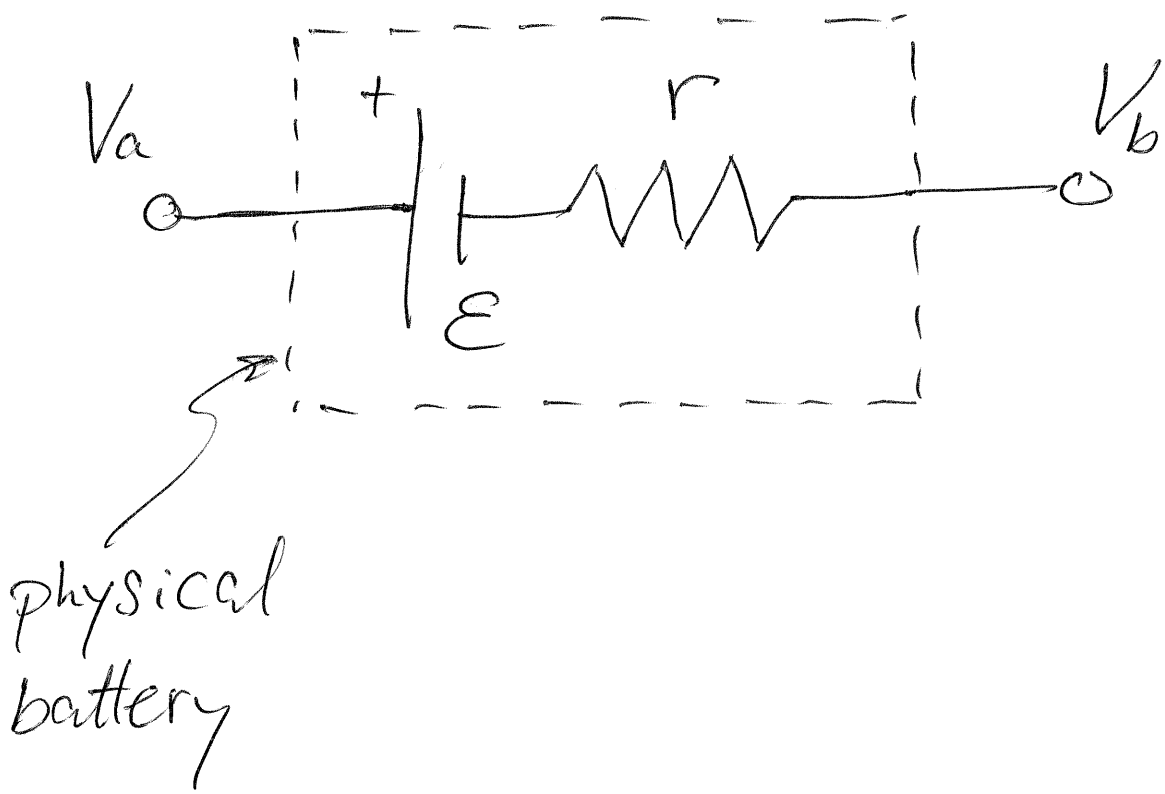
Lecture 16

In circuit theory the icon for a "pure emf" is

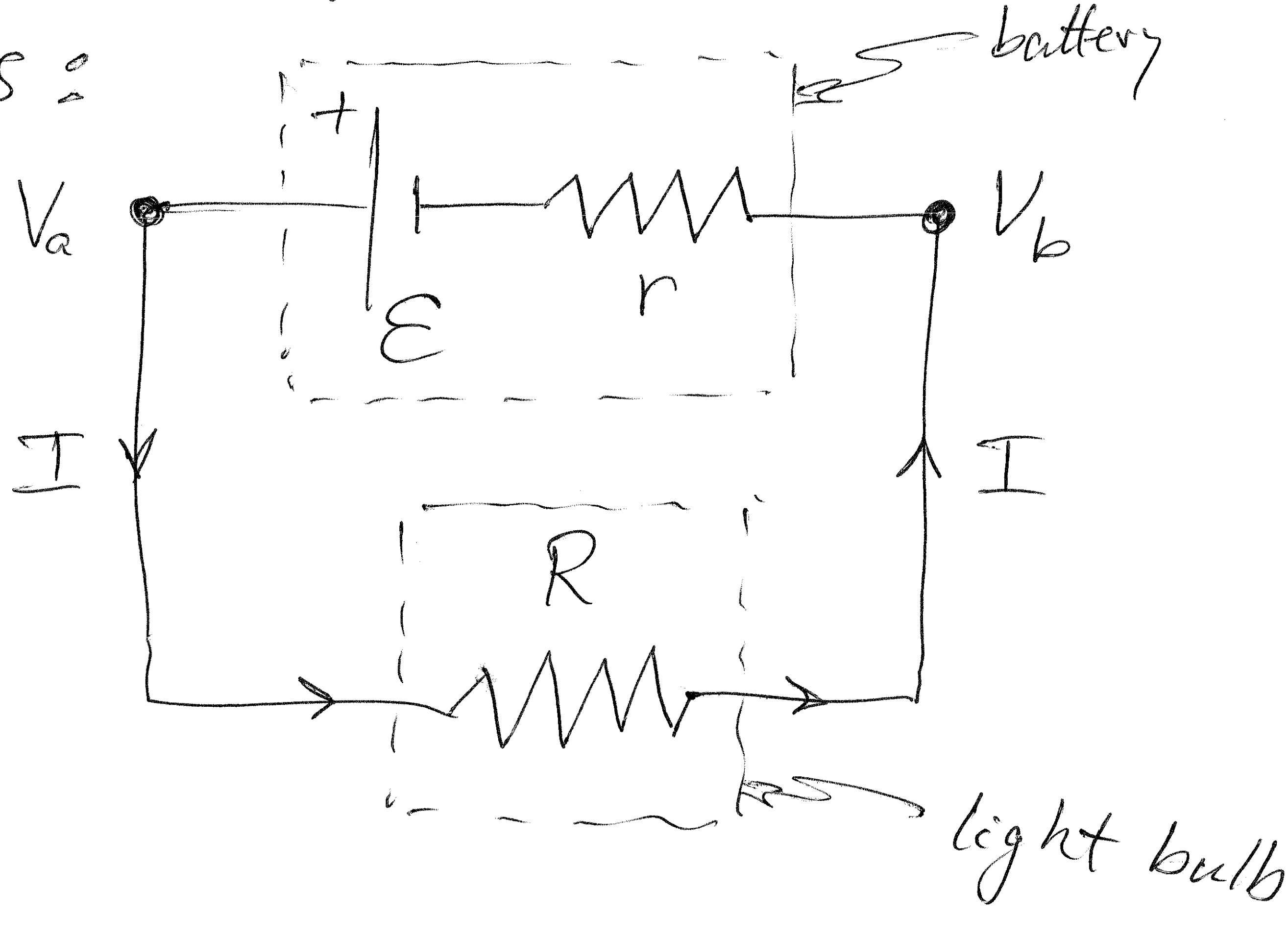


$$V_a - V_b = \mathcal{E}$$

where $\mathcal{E}$ is the energy per unit charge acquired by charges as they move from terminal **b** to terminal **a** (the side indicated by "+"). An actual battery will also have internal resistance in addition to the pure emf derived from chemical reactions. Both elements are represented schematically like this:



The potential $V_a - V_b$ across the terminals of the battery now also depends on the amount of current I flowing through the battery, since all of this current flows through the internal resistance r which has its own, current dependent, voltage difference. Attaching the battery to a light bulb we get a "complete" circuit like this:



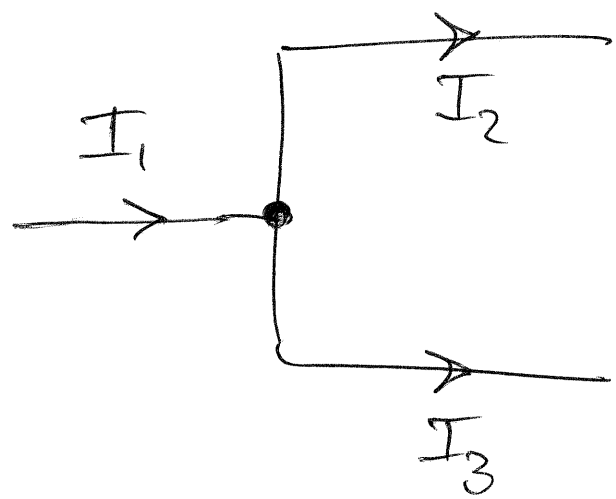
The light bulb is represented as a resistor R, since its action on the flow of charge is no different. In a light bulb the electrical energy is dissipated in a very small amount of material — a thin filament — so that it becomes very hot and emits radiation (electro-magnetic waves, actually!). The strong temperature variation of light bulbs has the effect that R is not as constant as it is in proper resistors.

We can relate the quantities in our circuit diagram by invoking two principles of physics:

(I) charge conservation

(II) energy conservation

Principle (I) is satisfied simply by the fact that we were careful to ensure that the same current I is flowing into and out of any component. If this were not the case, then by conservation of charge there would be an accumulation of positive or negative charge in the battery or light bulb and we would not have a steady-state. Later, when we study capacitors there <u>will</u> be charge accumulations and non-steady currents. Junctions involving three or more currents are also possible and are dealt with by imposing constraints on the <u>sums of currents</u>:



$$I_1 = I_2 + I_3$$

Principle (I) is called Kirchhoff's "junction rule" in electrical engineering.

Principle (II) asserts that when a charge is moved in the direction of the current around a loop in the circuit its energy is unchanged. The circuit is in a sense a model of a closed system, where all sources and losses of energy are accounted for. In our battery + light-bulb loop, for example, the current flow is counter-clockwise and a charge q gains energy $+\mathcal{E}q$ moving through the battery and ~~losses~~ loses energy $-V_r q$ and $-V_R q$ moving through the two resistors. The resistor voltages are current-dependent:

$$V_r = Ir \quad , \quad V_R = IR$$

Setting the net energy change equal to zero we find:

$$\mathcal{E} = Ir + IR$$
$$= I(r+R)$$

More generally (say we have several loops in our circuit) principle II asserts that it is possible to <u>consistently</u> assign a unique voltage to all the terminals, and for that to be true the <u>voltages around any loop must sum to zero</u>.

Electrical engineers call this Kirchhoff's "loop rule".

Notice that the voltage

across the physical battery $V = V_a - V_b$ is not the same as the "chemical" emf, $\mathcal{E}$. In fact, the voltage V is exactly the voltage across the bulb when a current I is flowing through its resistance R:

$$V = V_a - V_b = \text{battery terminal voltage}$$

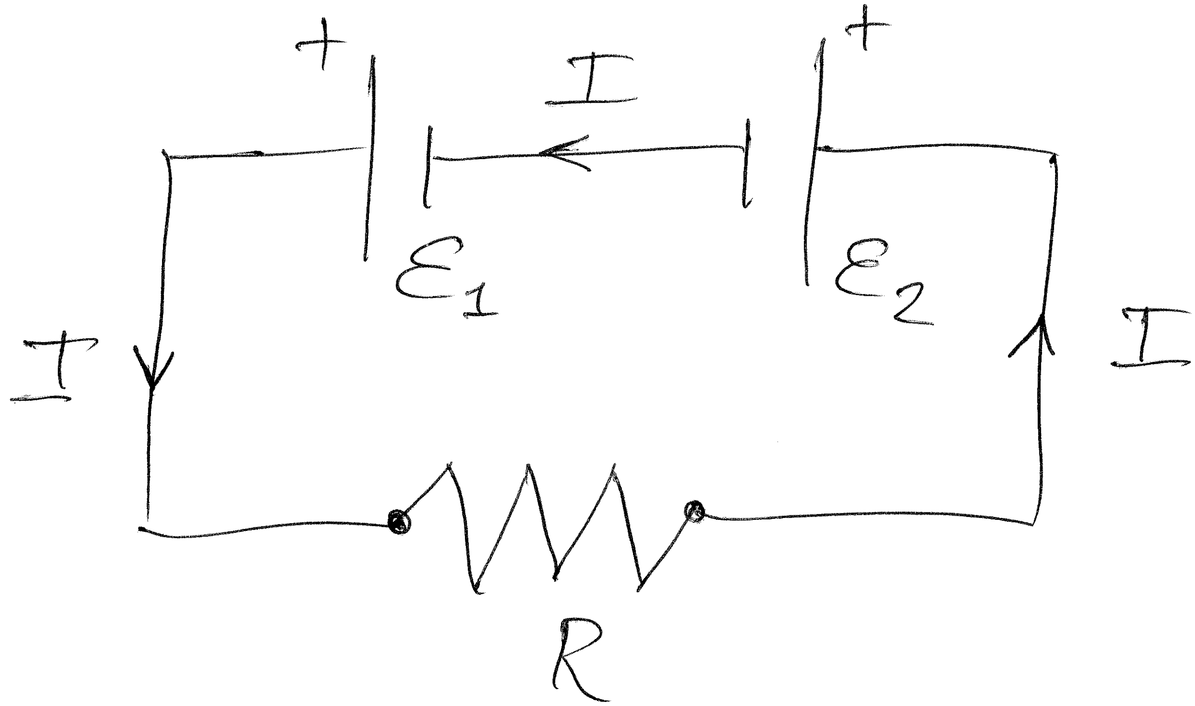
$$V = IR$$

$$= \left(\frac{\mathcal{E}}{r+R}\right) R = \left(\frac{R}{R+r}\right) \mathcal{E} < \mathcal{E}$$

The actual voltage applied to the bulb, V, will be significantly reduced relative to $\mathcal{E}$ when the internal resistance r of the battery is large.

The conservation of energy principle has an ~~close~~ equivalent expression in terms of power. We will illustrate this using the example of a circuit that shows a weak battery being "charged" by a stronger battery. Note that "charging" is a poor choice of verb in that the battery remains charge neutral while actually receiving energy/power.

We will omit explicit internal resistances to keep things simple:



loop rule: $\mathcal{E}_1 - \mathcal{E}_2 - IR = 0$

$\mathcal{E}_1 > \mathcal{E}_2 \Longrightarrow I > 0$

$$\mathcal{E}_1 = \mathcal{E}_2 + IR$$

mult. by I

$$\mathcal{E}_1 I = \mathcal{E}_2 I + I^2 R$$

The last equation is a power equation. It states that the power output of the strong battery (1) equals the power input to the weak battery (2) and the loss of power in the resistor.

Q: the weak battery gains power because current is flowing through it in the "wrong" direction. Describe the mechanism of energy gain in the case of a lead-acid battery.