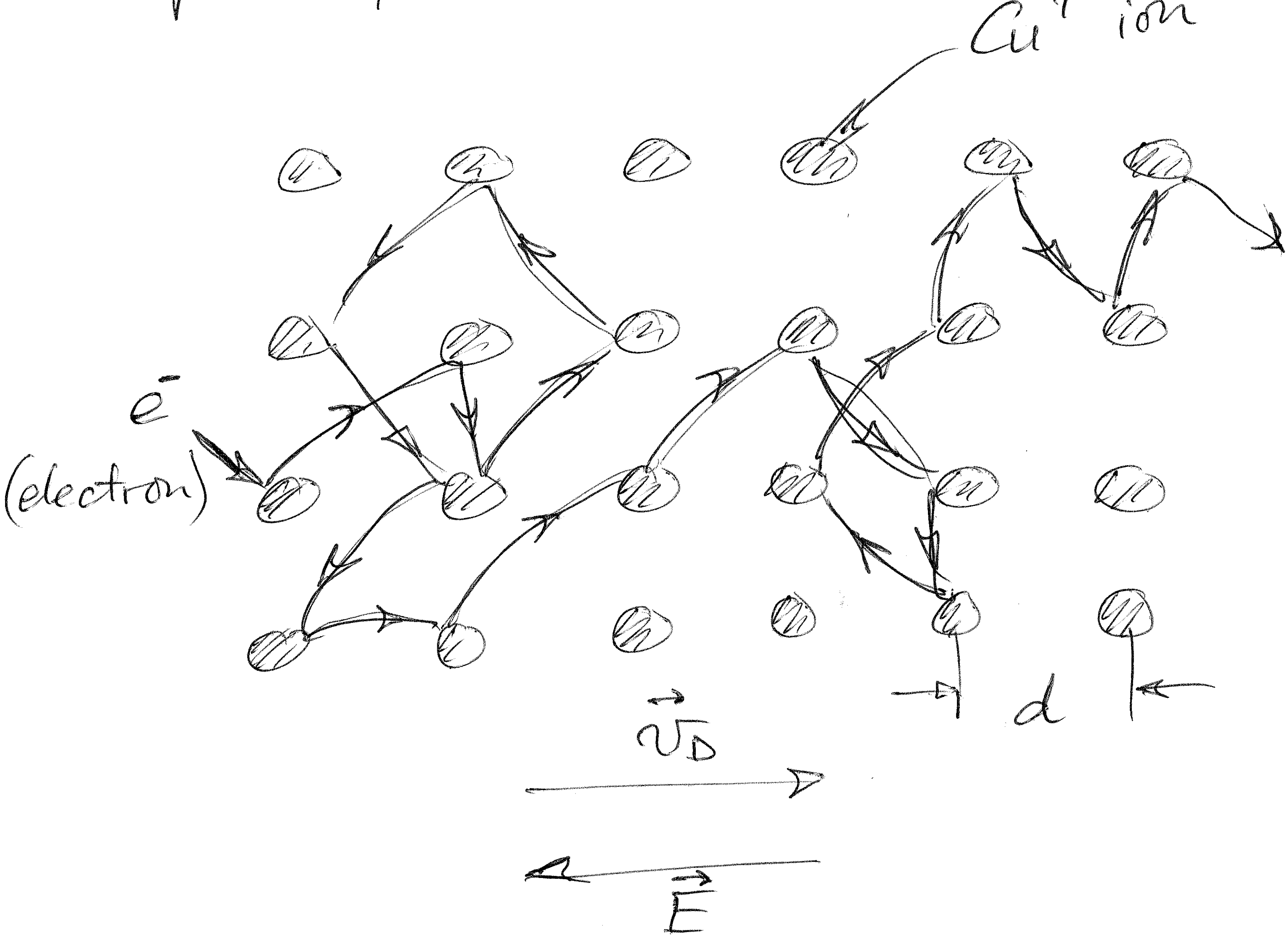
Lecture 14

We saw in the last lecture that the <u>thermal velocity</u> of an electron, $v_T \sim 10^5$ m/s, is <u>much</u> greater than the <u>drift velocity</u>, $v_D \sim 10^{-4}$ m/s, needed to explain the flow of charge in a typical conductor (1A in 1mm² cross-section Cu wire). Because thermal motion is random, and therefore isotropic, the actual motion of the conduction electrons in a metal is more like a "random walk", that goes nowhere on average, which has a very tiny drift superimposed:



Without the electric field the electron motion is purely random, scattering off into a random direction whenever it collides with one of the Cu^+ ions in the copper. With an electric field present, the ~~electrons~~ trajectories are slightly curved (accelerated) causing a slight drift opposite the direction of $\vec{E}$ (since electrons have negative charge). A key quantity is

$$\tau = \left(\begin{array}{l}\text{typical time between} \\ \text{collisions}\end{array}\right)$$

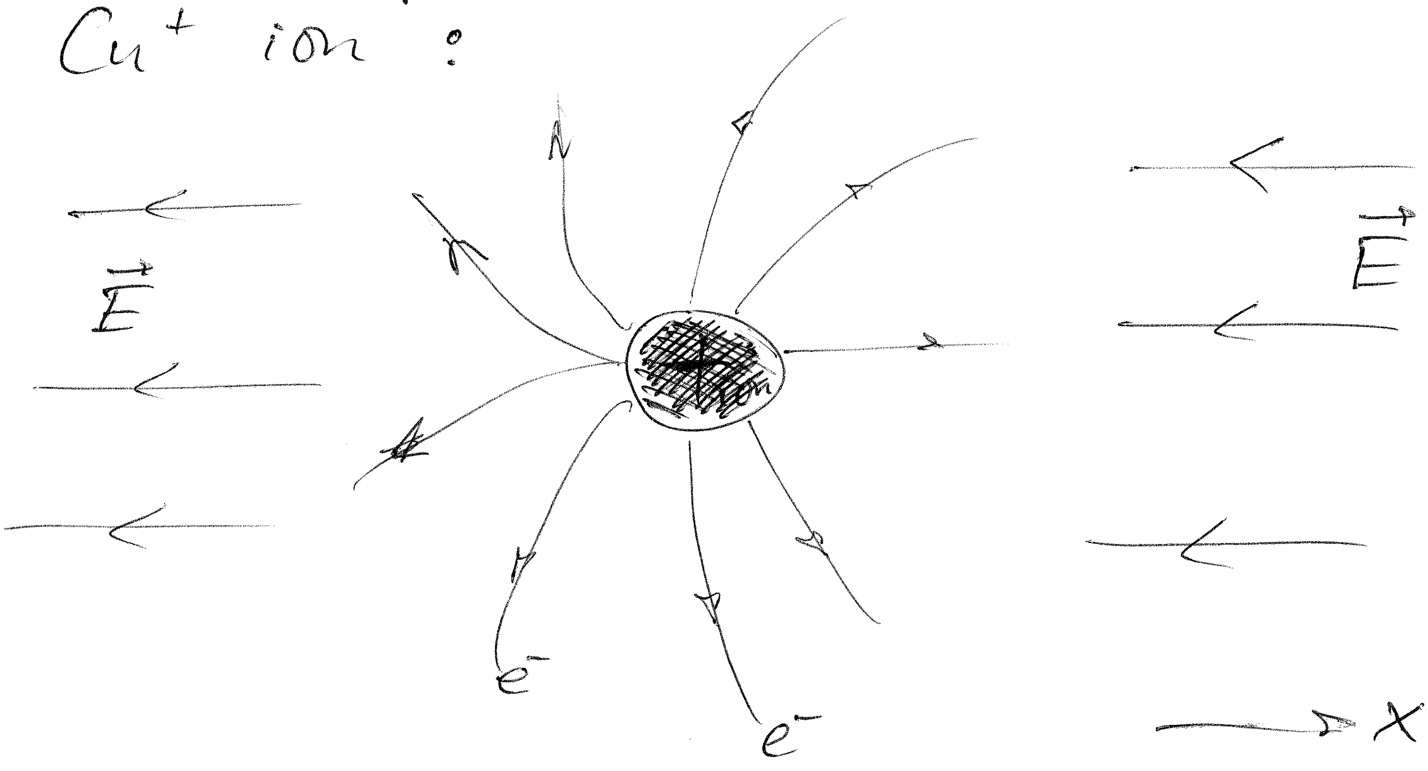
$$\simeq \frac{d}{v_T} = \frac{3 \times 10^{-10}\ m}{10^5\ m/s}$$

$$= 3 \times 10^{-15}\ sec.$$

We used for d in this estimate the distance between

Cu atoms.

How much drift do the electrons acquire during the short time τ when they are accelerated by the electric field? Here's a sketch of several random trajectories right after collision with a Cu^+ ion:

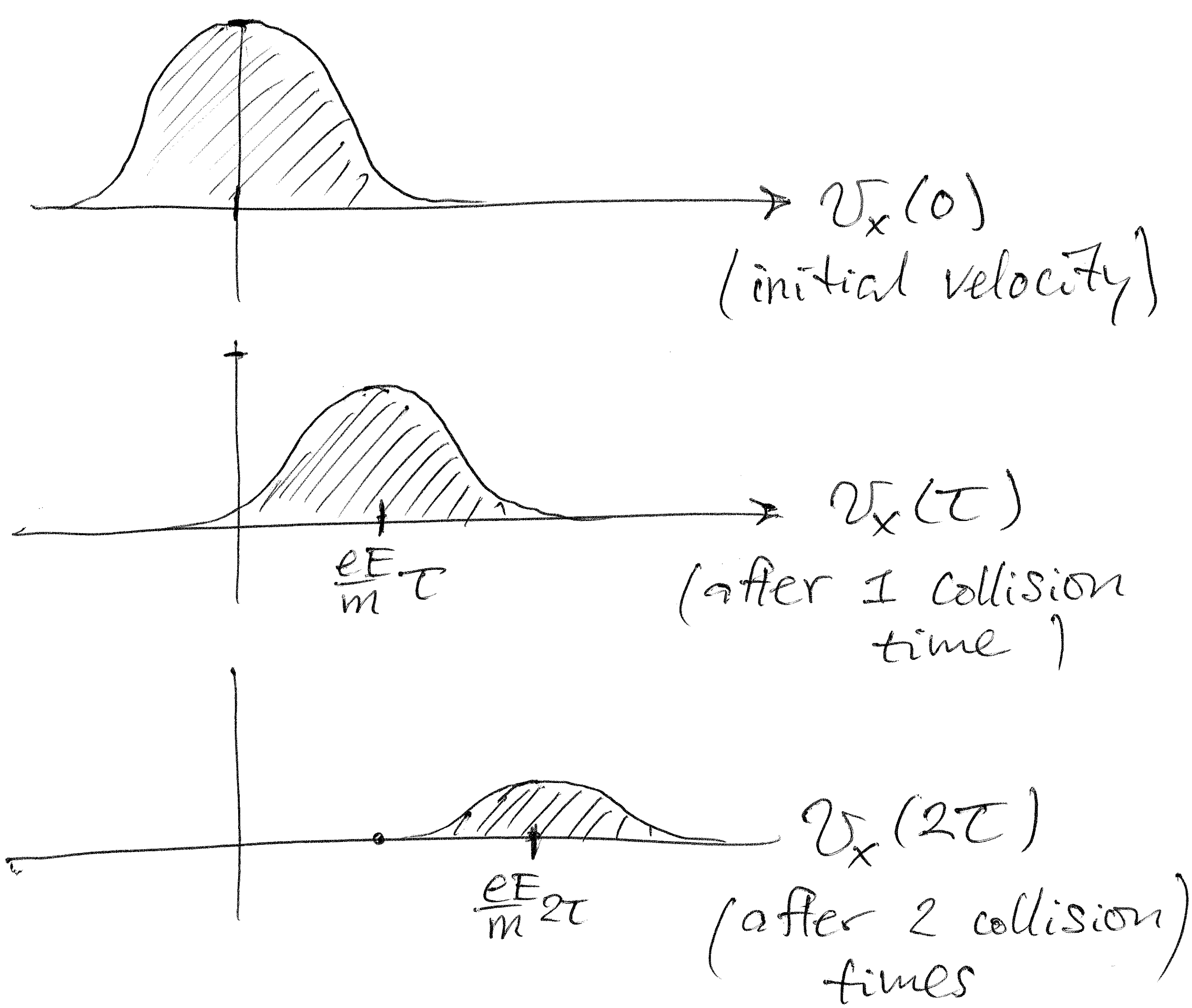


$$m a_x = F_x = (-e)(-E) = eE$$

$$v_x(t) = v_x(0) + a_x t = v_x(0) + \frac{eE}{m} t$$

(5)

Considering the random distribution of $v_x(0)$ due to the ion collision, we can make the following series of sketches of how the velocity distribution evolves with time:



The distributions have decreasing magnitude (area under curve) with time as fewer and fewer electrons have not been re-randomized by the next collision. The ~~me~~ mean of these distributions defines the drift velocity of the conduction electrons in aggregate:

$$v_D = \langle v_x \rangle \simeq \frac{eE}{m}\tau$$

This is consistent with the empirical law $v_D = \mu E$ and provides a microscopic estimate of the mobility:

$$\mu \sim \frac{e\tau}{m}$$

Let's use this estimate to predict the resitivity of Cu metal. From lecture 12,

$$\rho = \frac{1}{ne\mu},$$

so $$\rho \simeq \frac{m}{ne^2\tau}$$

$$= \frac{10^{-30}}{10^{29} \cdot (10^{-19})^2 \cdot 10^{-15}}$$

$$= 10^{-6}\ \Omega\,\mathrm{m}$$ (actual value 10^{-8} — not bad!)