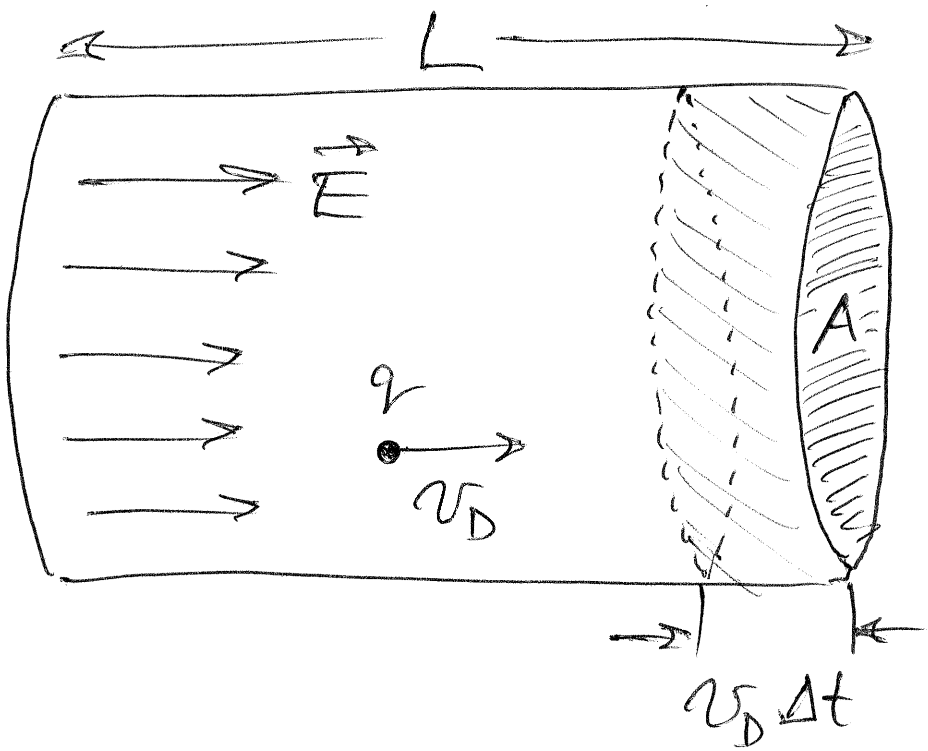
Lecture 12

We saw in the lightning rod demo that the presence of ions in the air — created at a sharp point where $E > 3 \times 10^6$ V/m — allowed a non-cataclysmic transfer of charge, an apparent steady-state flow. This is the next-best-thing to mechanical equilibrium: although there is motion, the state of motion is constant in time. A good example~~s~~ is particles falling in a uniform gravitational field that have reached "terminal velocity" as a result of friction.

We already know that we cannot have equilibrium when an electric field is in the presence of free charge. But we <u>can</u> have steady-state motion. Friction plays an important role here too: without it the free charge would continue to accelerate, never reaching a steady velocity. We will study the friction mechanism in more detail later. For now we'll assume it exists, so in conjunction with the electric field the free charges move with some "drift velocity" v_D.

Consider a conductor shaped as a cylinder (e.g. a short segment of wire):



$n =$ number density of charges q within the conductor

$L =$ length of cylinder

$A =$ cross-sectional area of cylinder

we assume the direction of the drift velocity is parallel/anti-parallel to $\vec{E}$ for positive/negative free charge (the drawing shows $q>0$). The shaded region of the cylinder corresponds to the total charge that leaves the cylinder during the time Δt:

ΔQ = total charge that leaves cylinder (or enters on other end) in time Δt.

$$\text{current} = I = \frac{\Delta Q}{\Delta t}$$

units: $\frac{C}{s}$ = "Ampere"

$$\Delta Q = \underbrace{(v_D \Delta t \cdot A)}_{\text{volume}} \underbrace{(n q)}_{\text{charge density}}$$

$$\frac{\Delta Q}{\Delta t} = n q v_D A = I$$

Georg Ohm discovered a simple law — Ohm's law — the content of which can be distilled to a proportionality between v_D and $|\vec{E}|$. We'll accept this as an "empirical law" for now and examine its origins later. The proportionality constant

μ is called the "mobility" of the free charge:

$$v_D = \mu |\vec{E}| = \mu \frac{V}{L}$$

We've expressed the uniform $|\vec{E}|$ in our cylinder in terms of the potential difference V between its ends and the length L.

Substituting this expression for v_D into our formula for current we find

$$n q \left(\mu \frac{V}{L}\right) A = I$$

$$\Rightarrow V = \underbrace{\left(\frac{1}{n q \mu}\right)\left(\frac{L}{A}\right)}_{R} I$$

R = "resistance"

units : $\frac{V}{I} = \frac{\text{Volt}}{\text{Amp}} =$ "Ohm" $= \Omega$

fund. units $= \frac{J/C}{C/s} = \frac{Js}{C^2}$

The "resistance" of our cylinder is the product of two parts:

$$R = \rho \left(\frac{L}{A}\right)$$

$\rho = \frac{1}{nq\mu}$ = resistivity

= material-dependent

$\frac{L}{A}$ = geometry dependent

unit of ρ : $R\frac{A}{L} = \Omega \cdot m$

The resistivity of a material can be very temperature-dependent. We will see in a demo that ρ can be dramatically <u>decreased</u> by <u>lowering</u> the temperature, in the case of one material, and <u>raising</u> the temperature in another!