Desert Island Project

Objective

Instead of a final exam, you will write a paper documenting your work on a mini-project. You have a choice of two types of project, detailed below. The point of the project is to convince the instructor that you are capable of working on a quantum system without outside help. Part of the assignment is proposing a system to study—something you should get started on as soon as possible! It might turn out to be something others have already looked at. That is not your concern, because you are working on a desert island. Also, it's important for your system to be simple enough that it is tractable mathematically—though approximations are allowed if well-motivated. Think of your system like the many "toy" systems you got training on when learning classical mechanics. The big difference, of course, is that your system is modeled with quantum mechanics.

Systems to avoid

Limit yourself to particles, or particles with hard constraints, like rigid bodies. No cats, quantum optics, many-body or condensed-matter systems. There are advanced courses for those. Also avoid systems inspired by the interpretation of quantum mechanics. This is not a philosophy course.

Real systems

The first type of system is a real one, where you could check your modeling and calculations in a lab (on a well-stocked island). Example: there is an excited state of the helium atom where both electrons are in the 2s orbital, not the 1s orbital. It's unstable to the decay into He^+ (where the electron is in the 1s orbital) and a free electron. What is the lifetime of this excited state?

Very precise calculations of the decay (and equally precise measurements of the lifetime) can be found in the literature. Your task is to work things out only to the lowest order in an approximation scheme. Spoiler: the lifetime is infinite when the Coulomb interaction between the electrons is neglected. *After* you've arrived at an answer for the lifetime that you are happy with, by all means search the literature to see how well you did! However, using the literature to get started is *not* allowed.

Toy systems

Toy systems are useful for understanding an effect because they are stripped of the irrelevant details in a physical system. Consider an electron incident on the surface of an insulator. To keep things simple, the insulator surface is perfectly flat and interacts with the electron through a step-like potential function $V(z)$, where z is the height above (or below) the surface. Classically there are two simple outcomes. When the incident electron's energy is below the potential inside the insulator it is reflected, when above, it is transmitted. In undergraduate courses this system is modeled with an abrupt (discontinuous) step, because the wave functions can be worked out with matching. That quantum model also predicts 100% reflection when the incident energy is below the classical threshold, but there is also some amplitude of reflection when the energy is above the classical threshold!

The undergraduate toy model may have left out an important detail: the effect of the step width w. If λ is the wavelength of the incident electron, the ratio $r = w/\lambda$ might be relevant (and was strictly zero in the undergraduate model). Is the classical result—100% transmission—recovered in the $r \to \infty$ limit of the quantum model? If so, it would be useful to know if the quantum correction decays as a power law or exponentially in r. To answer these questions you will probably want to make some choice for the functional form of the smooth step. And if you're lucky, the Schrödinger equation may have an analytic solution. Fortunately your desert island has an excellent math library (in-print and electronic), so that will not slow you down!

Guidelines

- Very brief project proposals submitted for approval, in print, by Monday, October 5.
- Approved projects will be posted on the course website. Try to not intrude on another student's project.
- Finished project submitted, in print, by Monday, December 7.
- AI allowed only for the *mechanics* of writing (spelling, LaTeX, formatting) and basic math facts (including advanced functions). *Not* allowed for finding prior work on your system, or organizing your analysis and presentation.
- Discussions with classmates is allowed and encouraged.