Assignment 8

Due date: Friday, October 29

Electric field of an oscillating charge

A classic source of electromagnetic radiation is a single point charge executing simple harmonic motion along one axis. Using what you learned about the electric field of a uniformly moving charge, and how to stitch together field lines when the motion changes abruptly, this exercise shows you how to obtain a good approximation of the radiation from a harmonic source.

First verify that the net flux of electric field through an enclosing surface is independent of the uniform motion by explicitly integrating the flux of the anisotropic field we obtained in lecture over the surface of a sphere centered on the instantaneous position of the charge.

Now consider a charge that moves with constant speed $c/2$ and switches direction every 2 nanoseconds so that it oscillates between two "turning points" approximately 1 foot apart. Sketch the electric field at the instant the charge is midway between the turning points and is moving to the right. Your sketch should extend out to a distance of 5 feet from the midpoint, and the axis through the turning points should be marked off in units of 1/2 foot. Show as circles the thin spherical shells (wherein the field is non-radial) separating domains where the field is radial. Be sure to indicate at what positions along your axis each circle is centered and where the radial field lines between them would converge (if they where extended all the way to the axis). You will be graded more on these quantitative characteristics than artistry. In fact, we recommend that you limit yourself to only 8 field lines per domain (with some attention to relativistic bunching).

Motion of a charge in a uniform electric field

A particle of mass m and charge q is at rest at the origin at time $t = 0$ in a frame where there is a uniform electric field of magnitude E in the positive x-direction.

Using the differential equation derived in lecture, for the momentum of a charged particle in an electric field, find the momentum (all components) of the particle at all times t (positive and negative).

Using your knowledge of special relativity and the momentum function you found above, determine the position of the particle at all times.