Assignment 7

Due date: Friday, October 22

Infinite grid of capacitors

Consider an infinite square grid, each node of which is connected to its four neighbors by a 1F capacitor. What is the effective (lumped) capacitance between a pair of adjacent nodes? In other words, if you wanted to store charge $+Q$ and $-Q$ at adjacent nodes, what potential difference would you have to apply between these nodes, say with a battery?

This problem is easier than it seems. First of all, you do *not* want to approach this as a fiendishly complex series/parallel combination circuit. The trick is to use symmetry. First consider the much more symmetric scenario, where you place only charge $+Q$ at one "central" node — the compensating negative charge will be at infinity (or evenly distributed about the perimeter, if you terminate the grid at some large distance from the central node). Using symmetry you should be able to work out exactly how charge is distributed *near* the central node, which is all you will need. Next consider an equally symmetric scenario, where only charge $-Q$ is placed, and at a node adjacent to the node in the first scenario. Finally, superpose the two scenarios and see what you get.

Energy and power in a simple RC circuit

Here are some energy-related problems for the series RC circuit we studied in lecture. The loop includes a pure EMF $\mathcal{E}$ and the capacitor is uncharged when the loop is closed at time $t = 0$.

By analyzing the circuit directly — not the solution to the differential equation — find the initial current, I_0, and the final voltage on the capacitor, V_∞.

From the solution to the differential equation, obtain the instantaneous power (i) provided by the EMF, (ii) consumed by the capacitor, and (iii) dissipated in the resistor. Check that these three functions of time are consistent with conservation of energy.

By integrating the power (i) above, obtain the total energy provided by the EMF. Compare this to the final energy stored in the capacitor, $CV_\infty^2/2$, and explain the discrepancy.