Assignment 4

Due date: Friday, September 24

Electric field inside an atom

The charge in a hydrogen atom comprises a point charge $+e$ at the origin, the proton, and a continuous charge distribution $\rho(r)$ associated with the electron "cloud" that varies with the distance r from the origin. In your first quantum mechanics course you will learn that $\rho(r)$ is derived from a wave function and has the form

$$\rho(r) = A\, e^{-r/a},$$

where a is an atomic length scale constructed from physical constants (about 3×10^{-11} m).

Determine the value of the constant A, in terms of a and the elementary charge e, so that the total charge in the electron cloud is $-e$.

Using symmetry and Gauss's law, determine the electric field (magnitude and direction) inside a hydrogen atom. Give limiting forms that apply when $r \ll a$ and also $r \gg a$. The answer $E(r) \to 0$ is not adequate for the second case: we want to know *how* it approaches zero.

Point charge in a uniform field

You already have good mental images of two electric fields: a uniform (constant) field and the field produced by a point charge. But suppose we place a point charge in a region where there already is a uniform field; what does that look like? This is the type of question for which a field-line drawing is very helpful. Without loss of generality, let the direction of the uniform field define the positive x-axis, so $\mathbf{E}_0 = E_0\,\hat{\mathbf{x}}$, $E_0 > 0$. We'll place our charge $q > 0$ at the origin.

Along the x-axis, by symmetry, the net electric field $\mathbf{E}(\mathbf{r})$ has only an x-component; the y- and z-components are zero. Make a plot (by hand) of $E_x(x)$ vs. x.

Now, by hand, make a field-line drawing of $\mathbf{E}(\mathbf{r})$ in the x-y plane. Here's a checklist for evaluating your drawing:

- Does it look like a uniform field far from the charge?
- Does it look like a point charge close to the charge?
- Do any field lines terminate or cross? Actually, this may happen in two instances where the direction of the field is not defined: on top of a point charge or at points where the electric field magnitude is zero. Both of these occur in your drawing.
- Are the arrows on the field lines consistent with the signs of sources/sinks and the direction of the field "at infinity"?

Determine the position of the point p on the x-axis where $\mathbf{E}(\mathbf{r}) = 0$. At p calculate the three partial derivatives $\partial E_x/\partial x$, $\partial E_y/\partial y$ and $\partial E_z/\partial z$. Add these to get the divergence of $\mathbf{E}(\mathbf{r})$ at p. Is your answer consistent with the local form of Gauss's law?

When a positive test charge is placed at p on the x-axis it is in equilibrium because the electric field there is zero. Check whether the equilibrium is stable or unstable by considering perturbations — not just in x — of the position about the equilibrium point.