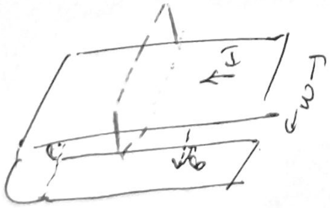
Assignment 12 Solutions

Parallel-sheet inductor

1. We can superpose the magnetic field from each ~~s~~ sheet to get the direction of the magnetic field inside. The field due to the top sheet points into the page above it and out of the page below it. The field from the bottom sheet points out of the page above it and into the page below it.

   Superposing both these fields, we see that $\vec{B}$ points out of the page in between the plates.

2. To find B, use the Amperian loop as shown

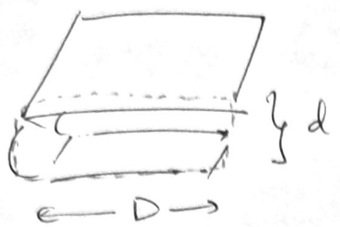


$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

$\Rightarrow B w = \mu_0 I$ (only the bottom side contributes)

$$B = \frac{\mu_0 I}{w}$$

3. Consider the curve C as shown:



$$\Phi_B = \oint_S \vec{B} \cdot d\vec{A}$$

$$= \frac{\mu_0 I}{w} \cdot dD$$

4.
$$\mathcal{E} = \frac{d\Phi_B}{dt} = L\frac{dI}{dt}$$

$$\Rightarrow \frac{\mu_0 \, dD}{w} \frac{dI}{dt} = L\frac{dI}{dt}$$

$$\Rightarrow L = \frac{\mu_0 \, dD}{w}$$

$\frac{dD}{w}$ has dimensions of length.

5.
$$\frac{1}{2} L I^2 = \frac{1}{2} \frac{\mu_0 \, dD}{w} I^2$$

$$\int \frac{B^2}{2\mu_0} dV = \left(\frac{\mu_0 I}{w}\right)^2 \cdot \frac{1}{2\mu_0} \cdot w \, dD$$

$$= \frac{1}{2} \frac{\mu_0 \, dD}{w} I^2$$

Thus $\frac{1}{2} L I^2 = \int \frac{B^2}{2\mu_0} dV$

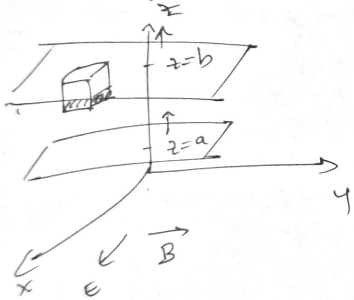
The moving slab of electromagnetic field

$\vec{E} = E\hat{x}$

$\vec{B} = B\hat{y}$

Since E and B are constants, their derivatives are zero. Therefore, $\vec{\nabla}\cdot\vec{E} = 0$ & $\vec{\nabla}\cdot\vec{B} = 0$ in the interior of the slab.

Now we need to check whether these are satisfied at the boundary. ~~Cuth~~ Consider a small cubic box fixed in space as shown. The planes cut through the cube at some time. The integrated equations,



$$\oint_{cube} \vec{E}\cdot d\vec{A} = 0$$

$$\oint_{cube} \vec{B}\cdot d\vec{A} = 0$$

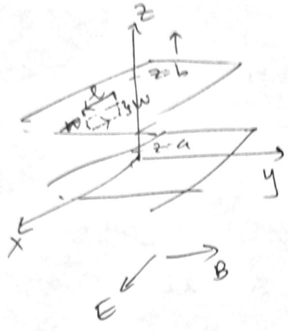
have to be satisfied.

$\vec{E}\cdot d\vec{A}$ is only non-zero for two of the faces of the cube, with $d\vec{A}$ pointing in the $\hat{x}$ & $-\hat{x}$ direction. While $\oint \vec{E}\cdot d\vec{A}$ for a single face changes with time, $\oint \vec{E}\cdot d\vec{A} = 0$ over the two faces because they have equal & opposite contributions. All the flux that enters the cube through one side exits through the other. ~~The~~

The argument is similar for $\oint \vec{B}\cdot d\vec{A}$ with the sides facing $\hat{y}$ & $-\hat{y}$ directions giving equal & opposite contributions. Thus, $\oint \vec{E}\cdot d\vec{A} = \oint \vec{B}\cdot d\vec{A} = 0$ at the boundaries.

We also see that $\vec{\nabla}\times\vec{E} = \vec{\nabla}\times\vec{B} = 0$ inside because E & B are constants. ~~At the boundaries, we need $\oint \vec{E}\cdot d\vec{l} = \oint \vec{B}\cdot d\vec{l}$~~

To check the validity of the curl equations at the boundaries, consider the loop shown, with length l and height w below the plane cutting through the $z=b$ plane at some instant of time. For this loop, we need



$$\oint_{loop} \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\Rightarrow -El = -\frac{d}{dt}(Bl\cdot w)$$

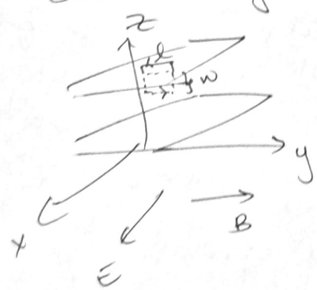
$\left\{ \Phi_B = \int \vec{B}\cdot d\vec{A} = B \times lw \right.$ since only the height w below the plane has $\vec{B} \neq 0$ $\left.\right\}$

$$\Rightarrow E = B\dot{w}$$

$$\Rightarrow E = B\dot{b} \quad \text{----} \ (1) \quad (\dot{w} = \dot{b}$$

At $z=a$, $E = B\dot{a}$

Similarly for the loop below, we need



$$\oint \vec{B}\cdot d\vec{l} = \frac{1}{c^2}\frac{d\Phi_E}{dt}$$

$$\Rightarrow Bl = \frac{1}{c^2}\frac{d}{dt}(E\,l\,w)$$

$$\Rightarrow B = \frac{E}{c^2}\dot{b} \quad \text{.....} \ (2)$$

At $z=a$, $B = \frac{E}{c^2}\dot{a}$

~~Eanate~~ From (1) & (2), we get $\dot{b} = c$. Similarly, $\dot{a} = c$

and $\frac{E}{B} = c$

The slab velocity is in the z-direction and has to point in the direction of $\vec{E} \times \vec{B}$

$\vec{E} = E\hat{x}$
$\vec{B} = B\hat{y}$

$\Rightarrow \vec{S} = \frac{1}{\mu_0} EB\hat{z}$, which points in the direction of velocity of the slab.