Comment on "Quasicrystals: A New Class of Ordered Structures"

In an interesting Letter, Levine and Steinhardt[1] have proposed that a three-dimensional (3D) generalization of the Penrose tiling (3DPT) could explain the (noncrystallographic) icosahedral symmetry of the diffraction pattern reported by Shechtman *et al.*[2] Here we point out that both the symmetry and the sharpness of the Bragg peaks appear to be unaffected by the introduction of a very general kind of disorder.

Figure 1(a) shows an aggregate of the two rhombic tiles (heavy lines) making up the analogous 2DPT. If we observe only the steric constraints imposed by the shape of the tiles, the number of ways of tiling the interior of an aggregate almost certainly grows as c^N where N is the number of tiles. Different tilings, for example, can be generated by the local rearrangement shown in Fig. 1(b). In contrast, the 2DPT does in fact have zero configurational entropy because the tiling configurations are severely restricted by the so-called "matching rules." Perhaps a natural expression of these rules is in terms of families (in this case five) of quasiperiodically spaced lines as discussed by Levine and Steinhardt.[1] When this pattern of lines is superimposed on the tiling [light lines in Fig. 1(a)], each rhombus of the two types is decorated in exactly the *same way*. Thus if one builds upon an aggregate with such decorated rhombi, it will be necessary to check whether the continuation of the lines within a rhombus is consistent with the already existing pattern of quasiperiodic lines. Figure 1(a) shows an inconsistency between two distant tiles marked A and B. While this conflict can be resolved by replacing tile B with the other rhombus, it points out the infinite-range nature of the constraints. This suggests that a 3DPT generated by a realistic growth process would have matching-rule violations and thus be disordered.

Strong arguments can be given to support the conclusion that configurationally disordered 3DPT's would still have an icosahedral point diffraction pattern.[3] A convenient construction of the more general (disordered) class of tilings involves the projection of the lattice analog of a 3D hypersurface embedded in 6D.[4] The ideal 3DPT corresponds to a *hyperplanar* surface which is "roughened" by disorder. Similarly, the diffraction pattern is obtained by projecting 6D reciprocal lattice vectors $G=(\mathbf{G},\mathbf{G}_\perp)$ where $\mathbf{G}$ refers to the physical momentum of a peak while $\mathbf{G}_\perp$ is a three-vector quantity that determines its intensity. The behavior of intensities with $|\mathbf{G}_\perp|$ appears to be a simple indicator of disorder. For example, if the hypersurface fluctuations were Gaussian (as would be expected if matching rules were ignored), then intensities would decay as $\exp(-c\mathbf{G}_\perp^2)$ in contrast to the power-law decay predicted for the ideal 3DPT.

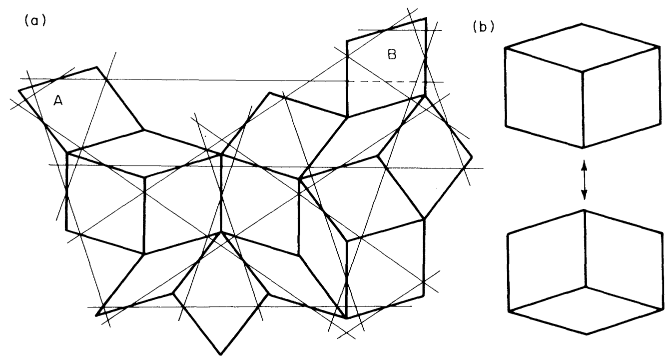

FIG. 1. (a) Two-dimensional Penrose aggregate showing quasiperiodically spaced lines (matching rules). (b) A rearrangement of three tiles.

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