Lecture 7

time reversal symmetry:

$t' = -t$

↑ time coordinate in time-reversed world

Newtonian dynamics (fundamental basis of mechanical systems: slinky, ...)

$$m \frac{d^2 x(t)}{dt^2} = F(x(t))$$

$$\frac{dx}{dt} = -\frac{dx}{dt'}, \quad \frac{d^2x}{dt^2} = +\frac{d^2x}{dt'^2}$$

$$m \frac{d^2 x(t')}{dt'^2} = F(x(t'))$$

form of equation is <u>unchanged</u>

$$c\frac{\partial\psi}{\partial t} = -c\frac{\partial\psi}{\partial t'}$$

changes in time-reversed world

$(D\cos\omega t)\psi$: fails test of time-translation symmetry

$t' = t - t_0$ (shifts arbitrary origin of time)

$$(D\cos\omega t)\psi = D\cos(\omega t' + \phi)\,\psi$$

$$\phi = \omega t_0$$

changes with time translation.

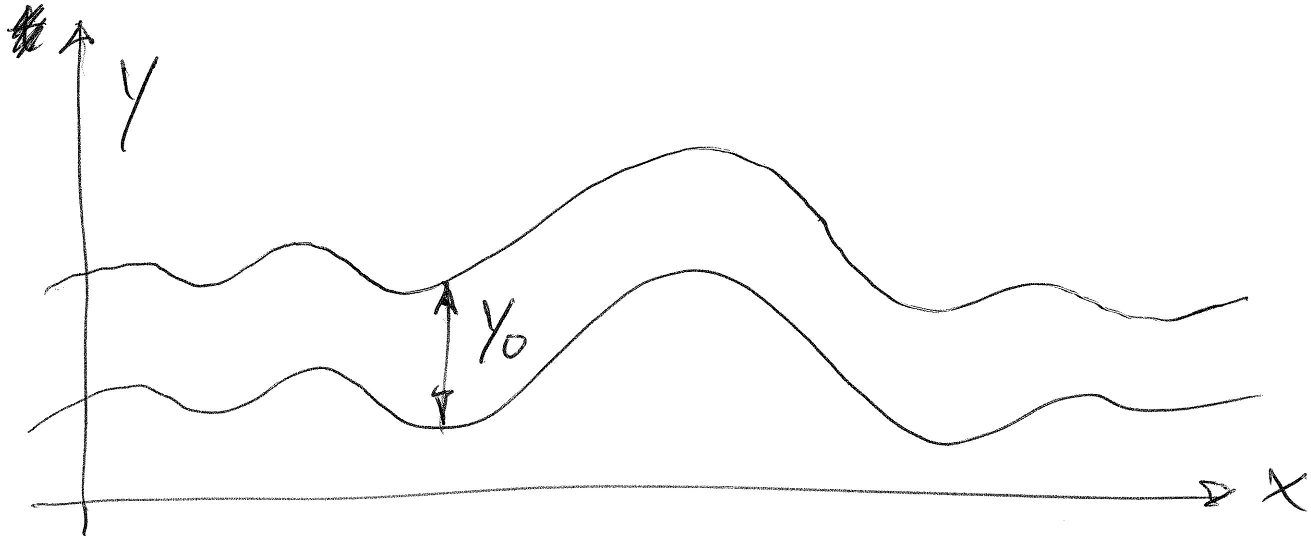
$E\psi$: changes when wave-amplitude is translated.

$\psi' = \psi - \psi_0$:

$$E\psi = E\psi' + E\psi_0 \neq E\psi'$$

transverse oscillations of elastic string have this symmetry:

$$\psi(x,t) = y(x,t)$$



$y' = y - y_0$

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2} \Longrightarrow \frac{\partial^2 y'}{\partial t^2} = v^2 \frac{\partial^2 y'}{\partial x^2}$$

form unchanged

4th derivative terms consistent with :

- locality
- linearity
- time-reversal
- time-translation
- amplitude translation

$$\frac{\partial^2 \psi}{\partial t^2} + T^2 \frac{\partial^4 \psi}{\partial t^4} = v^2 \left(\frac{\partial^2 \psi}{\partial x^2} + L^2 \frac{\partial^4 \psi}{\partial x^4} \right)$$

T = constant with dimensions of time

L = constant with dimensions of length

Express solution in terms of function that takes <u>dimensionless</u> arguments :

$$\psi(x,t) = F(\tilde{x}, \tilde{t})$$

$$\tilde{x} = x/\lambda \quad , \quad \tilde{t} = t/\tau$$

λ = spatial scale of solution

τ = temporal scale of solution

$$\frac{\partial F}{\partial t} = \frac{\partial \tilde{t}}{\partial t}\frac{\partial F}{\partial \tilde{t}} = \frac{1}{\tau}\frac{\partial F}{\partial \tilde{t}}$$

$$\frac{\partial^2 F}{\partial t^2} = \frac{1}{\tau^2}\frac{\partial^2 F}{\partial \tilde{t}^2}$$

$$\frac{\partial^4 F}{\partial t^4} = \frac{1}{\tau^4}\frac{\partial^4 F}{\partial \tilde{t}^4}$$

similarly for $x, \tilde{x}$ derivatives

Equation for F:

$$\frac{1}{\tau^2}\left(\frac{\partial^2 F}{\partial \tilde{t}^2} + \frac{T^2}{\tau^2}\frac{\partial^4 F}{\partial \tilde{t}^4}\right) = \frac{v^2}{\lambda^2}\left(\frac{\partial^2 F}{\partial \tilde{x}^2} + \frac{L^2}{\lambda^2}\frac{\partial^4 F}{\partial \tilde{x}^4}\right)$$

(7.6)

May __neglect__ 4th derivative terms if $T^* \ll \tau$, $L \ll \lambda$

$\rightarrow$ time/length scales of solution much larger than intrinsic time/length parameters (T/L) in wave equation