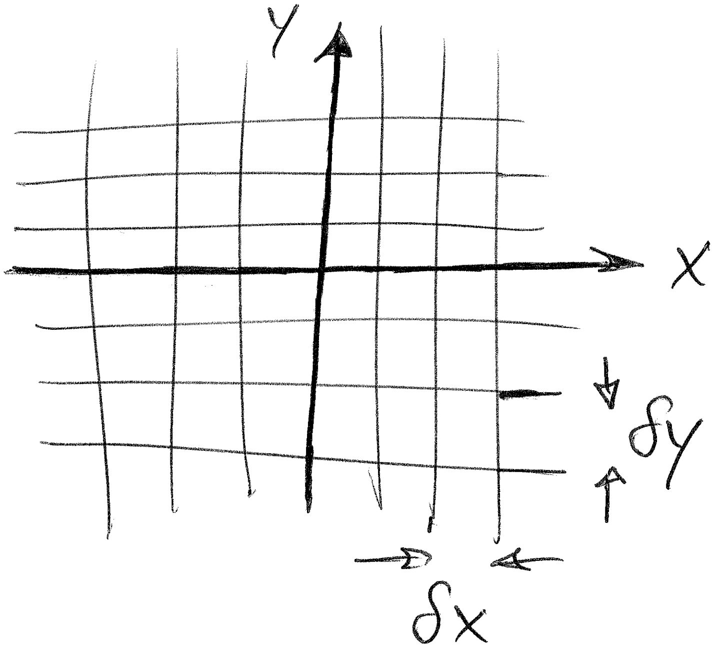
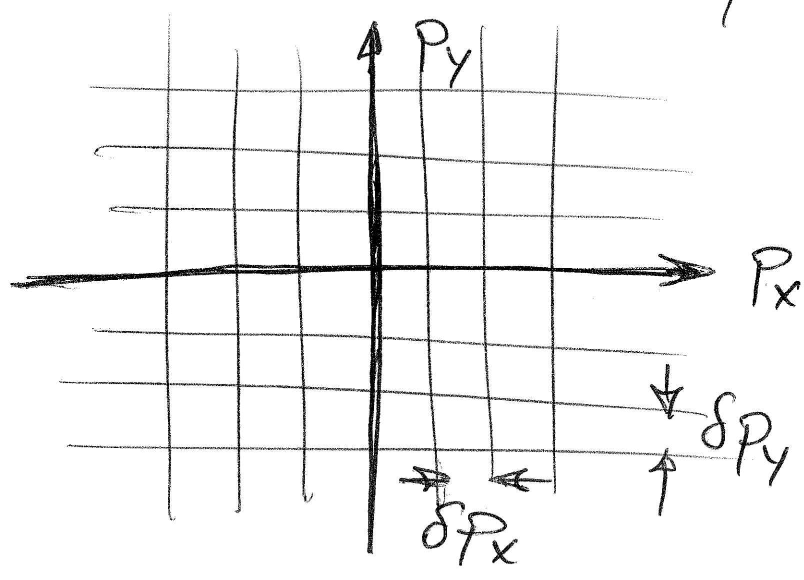
Lecture 34

The demo of the particle moving in the 2D potential is very convincing of the counter intuitive fact, that the particle spends just as much time in the region where the potential is low and it is moving fast! This is difficult to understand at the level of mechanics and motivates a new approach.

We'll start by learning how to enumerate microstates. Position states don't pose much of a problem: we simply subdivide space with a fine regular grid:



We also need a grid of momentum states to completely specify the microstate. [We choose momentum over velocity because it plays a more fundamental role in physics. This choice, however, makes no difference in classical physics.]



How do we introduce the constraint that our micro-states have a certain fixed amount of energy? There are good reasons we ~~only~~ want to specify a small range of energies δE rather than a precise value. In a sense this is already implied by our discretization of position and momentum. More fundamentally, we know from quantum mechanics that the energy will always be imprecisely defined when the system is observed over a finite time. We therefore include all microstates whose energy lies in the range E to $E+\delta E$.

All the quantities δx, δy, (34.4)

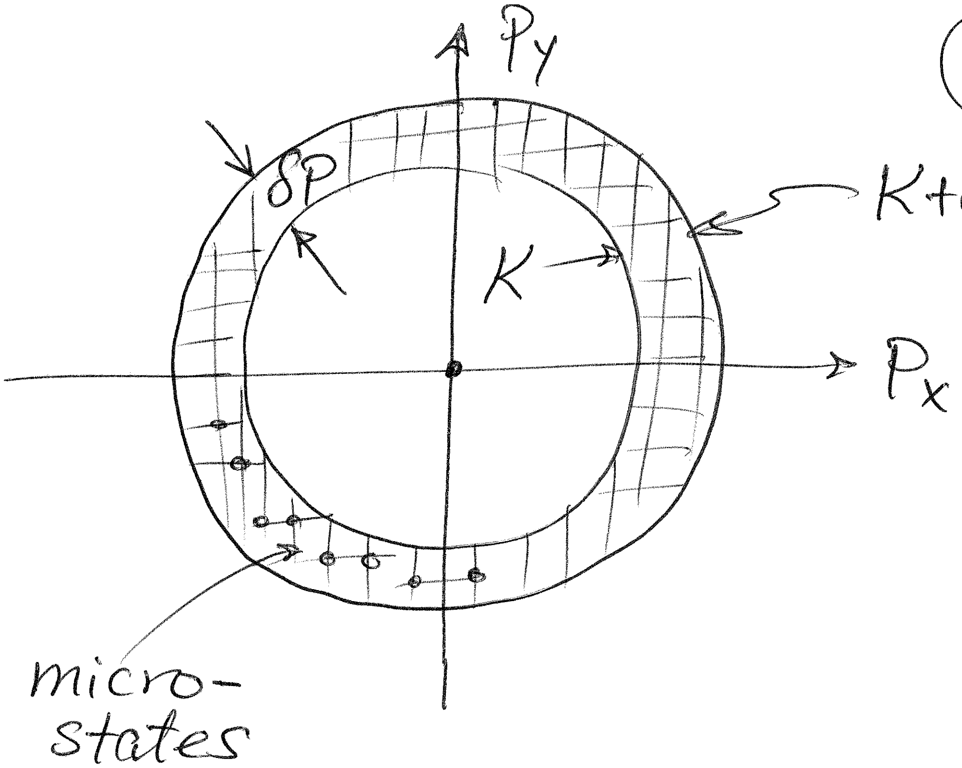
δp_x, δp_y, δE are fixed once and for all; they can be as small as we want without affecting our conclusions.

Let's return to our particle in the 2D potential. Suppose it is at a position microstate where its kinetic energy is K. The region in momentum space it can access is then an annulus with inner and outer radii given by

$$K = \frac{p^2}{2m}$$

$$K + \delta E = \frac{(p+\delta p)^2}{2m}$$

(34.5)



We can expand the second equation for $\delta p \ll p$ to find

$$\delta E = \frac{p\,\delta p}{m}.$$

From this we determine the thickness δp of the annulus. The number of microstates inside the annulus is its area divided by the area of one square in our grid:

$$\begin{pmatrix}\text{num. momentum micro-}\\ \text{states with energy in range}\\ K \text{ to } K+\delta E\end{pmatrix}$$

$$= \frac{(\text{annulus area})}{\delta p_x \, \delta p_y}$$

$$= \frac{2\pi p \, \delta p}{\delta p_x \, \delta p_y} = \frac{2\pi m \, \delta E}{\delta p_x \, \delta p_y}$$

This is a <u>constant</u>, independent of K, and therefore the same at all position states (high or low potential). Our basic hypothesis states that all micro states are visited equally over the course of time. But if the number of momentum states is the same at all position states, then the position states will be visited equally.

What changes when the particle moves in 3D?

2D annulus $\longrightarrow$ 3D thin spherical shell

$$2\pi p\,\delta p \longrightarrow 4\pi p^2\,\delta p$$

$$= 4\pi p \; p\,\delta p$$

$$\left.\begin{aligned} K &= \frac{p^2}{2m} \\ \frac{p\,\delta p}{m} &= \delta E \end{aligned}\right\} \longrightarrow \quad = 4\pi \sqrt{2mK}\; m\,\delta E$$

$$\begin{pmatrix}\text{num. momentum} \\ \text{microstates} \\ \text{in 3D}\end{pmatrix} \propto \sqrt{K} \propto v$$

So in 3D there are <u>more</u> momentum microstates at positions where K is large. Since $\sqrt{K} \propto v$ (particle speed), the particle will actually spend more time in places where it's going fast!!