# Lecture 2

A very general solution of the wave equation:

$$\theta(x,t) = f(x-ct)$$

$f$ = arbitrary function

$c$ = constant, to be determined

**Check** $$\frac{\partial\theta}{\partial x} = f'(x-ct)$$

( $f'(\ )$ = derivative of $f$ with respect to its argument )

$$\frac{\partial^2\theta}{\partial x^2} = f''(x-ct)$$

$$\frac{\partial\theta}{\partial t} = f'(x-ct)\cdot(-c)$$

$$\frac{\partial^2\theta}{\partial t^2} = f''(x-ct)\cdot(-c)^2 = \frac{\partial^2\theta}{\partial x^2}(-c)^2$$

Solves wave equation provided

$$c = \pm v$$

Both cases may be combined, additively, because wave equation is linear.

Most general solution:

$$\Theta(x,t) = f(x - vt) + g(x + vt)$$

$f, g$ : arbitrary functions

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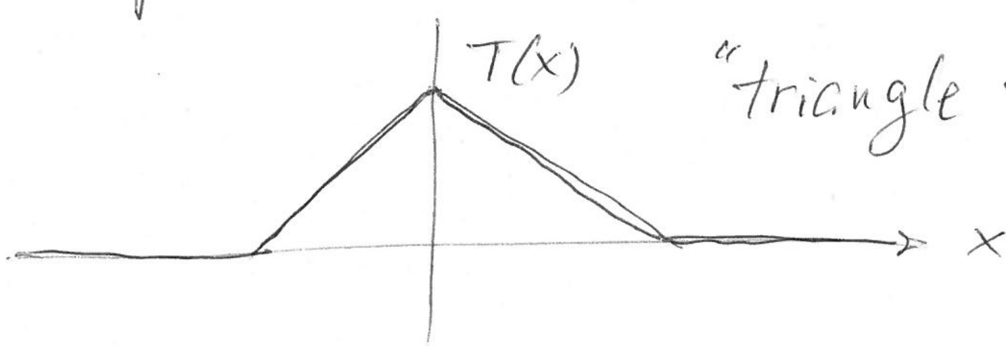
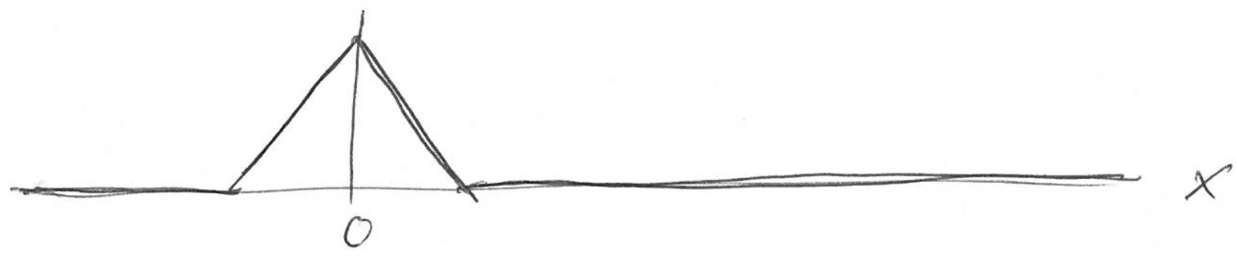
Example: $f = T$, $g = 0$

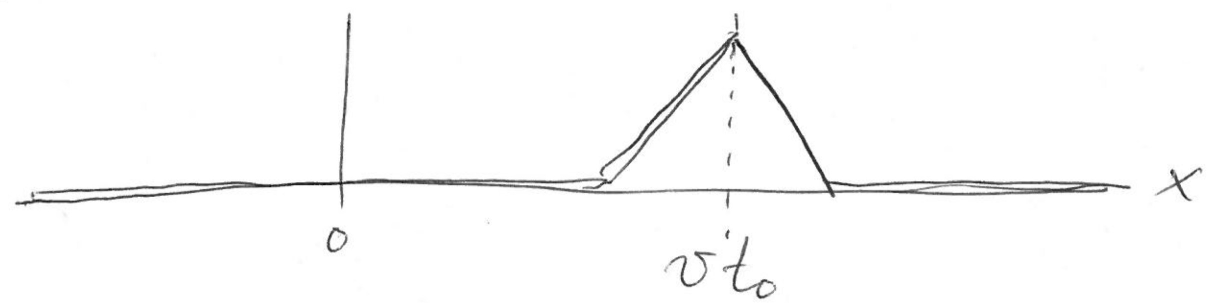



$$\Theta(x,t) = T(x - vt)$$

$\theta(x,0)$




$\theta(x,t_0)$




solution = right-moving "pulse"

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The general solution has a <u>redundancy</u>:

$$\tilde{f} = f + c \quad \& \quad \tilde{g} = g - c$$

describe <u>same solution</u> as $f$ & $g$, for any $c$. Notice that

$$\tilde{f}(0) - \tilde{g}(0) = f(0) - g(0) + 2c,$$

so we can choose $c$ so that $\tilde{f}(0) - \tilde{g}(0) = 0$.

This is convenient for what we do next.

# General initial conditions

Must specify, for all $x$, both

$\Theta(x,0) \leftarrow$ initial "positions"

and $\dot{\Theta}(x,0) \leftarrow$ initial "velocities"

Apply to general solution with property $f(0) = g(0)$ :

$$\begin{pmatrix}\text{initial pos.}\\ \text{function}\end{pmatrix} = p(x) = \Theta(x,0) = f(x) + g(x)$$

$$\begin{pmatrix}\text{initial vel.}\\ \text{function}\end{pmatrix} = q(x) = \dot{\Theta}(x,0) = -v f'(x) + v g'(x)$$

Solve for $f$ & $g$ in terms of $p$ & $q$:

$$\int_0^x q(x)\,dx = -v\big(f(x) - f(0)\big) + v\big(g(x) - g(0)\big)$$

$$= -v f(x) + v g(x)$$

(since $f(0) = g(0)$)

$$v p(x) - \int_0^x q(x)\,dx = 2v\, f(x)$$

$$v p(x) + \int_0^x q(x)\,dx = 2v\, g(x)$$

$$f(x) = \frac{1}{2} p(x) - \frac{1}{2v} \int_0^x q(x)\,dx$$

$$g(x) = \frac{1}{2} p(x) + \frac{1}{2v} \int_0^x q(x)\,dx$$

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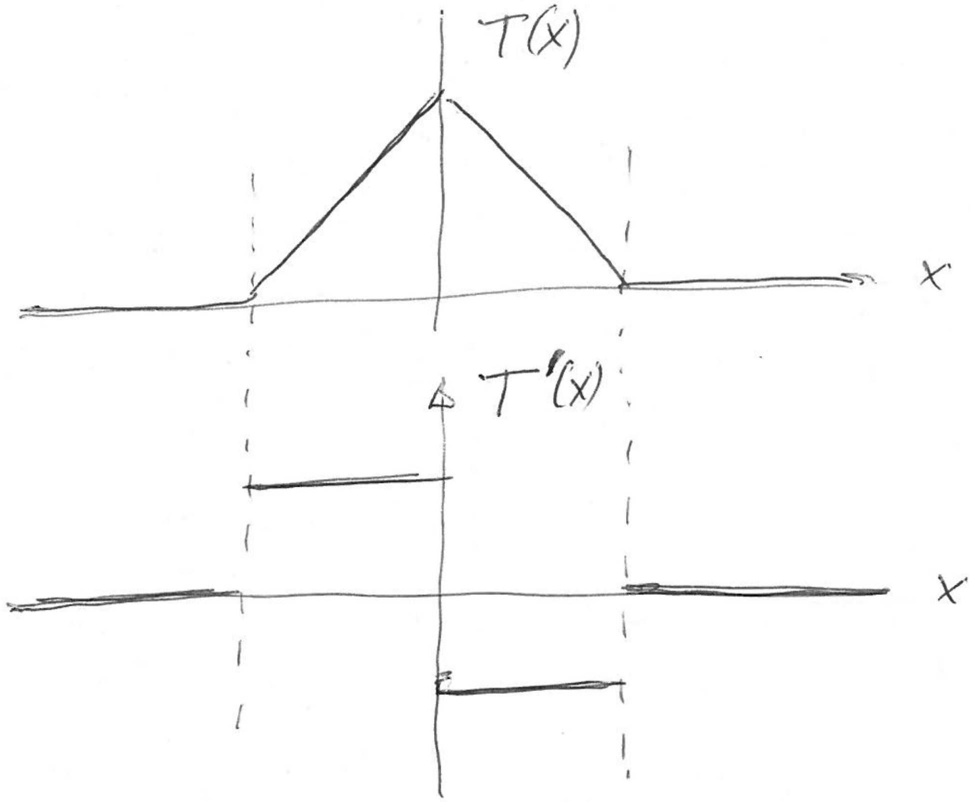
Example: $q(x) = 0$ (torsional oscillator is released from rest)

$$\theta(x,t) = f(x - vt) + g(x + vt)$$

$$= \frac{1}{2} p(x - vt) + \frac{1}{2} p(x + vt)$$

equal right & left-moving pulses

Specialize further: $p = T$




$$\dot{\Theta}(x,0) = -\frac{v}{2}T'(x) + \frac{v}{2}T'(x) = 0$$ ✓

$\Theta(x, t_0)$