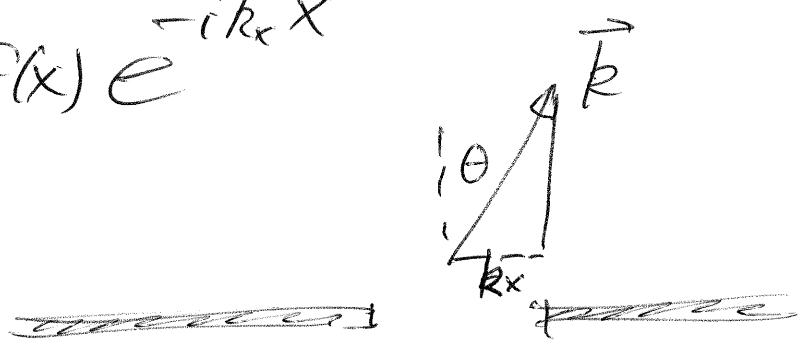
Lecture 26

Let's take another look at the diffracted wave amplitude:

$$\hat{f}(k_x) = \int_{-\infty}^{\infty} \frac{dx}{2\pi}\, f(x)\, e^{-ik_x x}$$

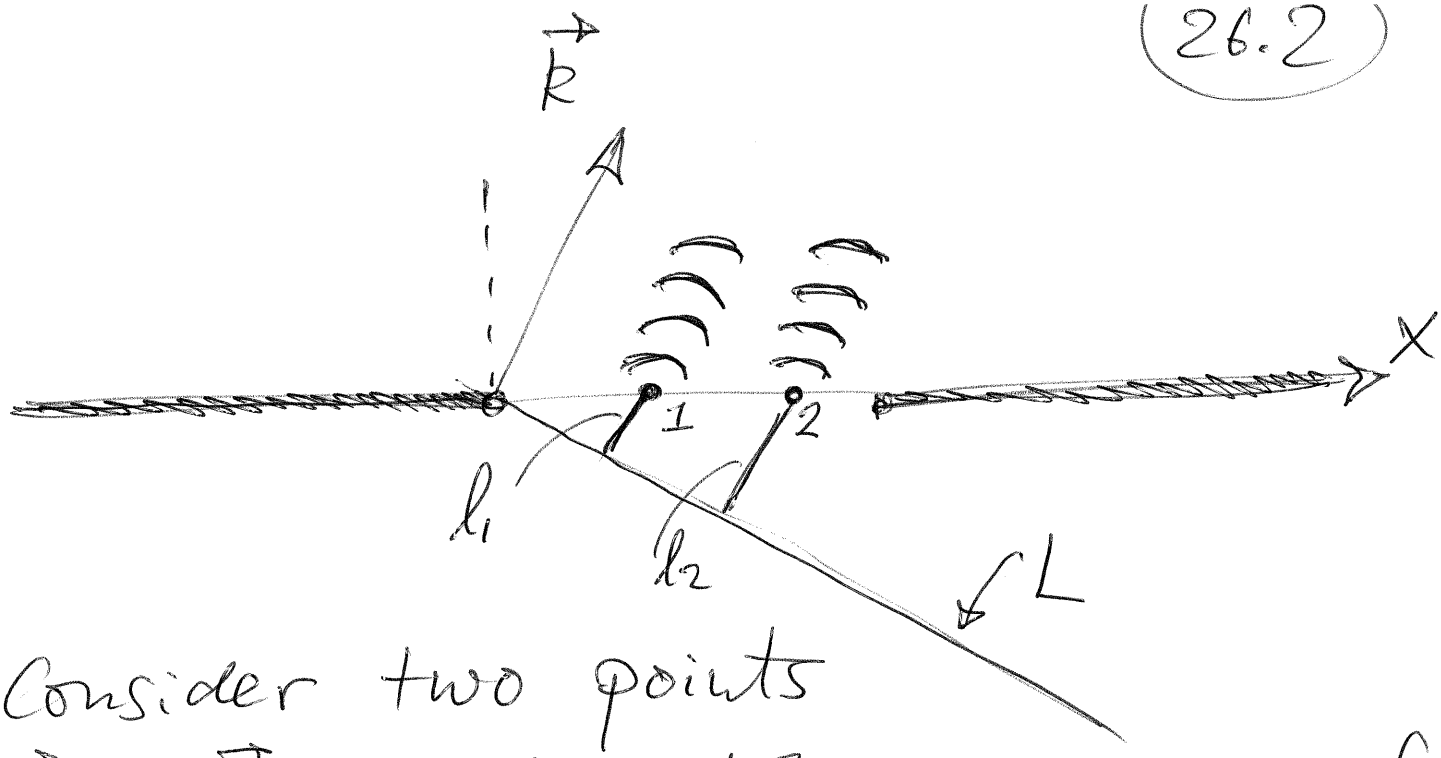


$$k_x = \left(\frac{2\pi}{\lambda}\right) \sin\theta$$

$$\hat{f}(\theta) = \int_{-\infty}^{\infty} \frac{dx}{2\pi}\, f(x)\, e^{-i2\pi\left(\frac{x\sin\theta}{\lambda}\right)}$$

The geometric interpretation of this is the <u>Huygens</u>- <u>Fresnel</u> <u>principle</u>.

We will illustrate the principle for the case where $f(x)$ describes a simple slit, but the principle is completely general.



Consider two points $\vec{r}_1$ & $\vec{r}_2$ in the slit as <u>sources</u> of waves. They are "in phase" or "in synchrony" because we think of them as oscillating in response to the incident plane wave from below, which has the same phase everywhere along the slit. However, if we ask how these point sources contribute to a wave with vector of propagation $\vec{k}$, then they will have <u>different</u> phase origins since the distances l_1 & l_2 to a common phase origin (line L) are different:

$$\cos(\vec{k}\cdot\vec{r}) = \cos(0) \quad \text{for } \vec{r} \text{ on } L$$

$$\psi_1 = \cos(\vec{k}\cdot\vec{r} - \phi_1) \quad \phi_1 = \text{phase origin of source 1}$$

$$\psi_2 = \cos(\vec{k}\cdot\vec{r} - \phi_2) \quad \phi_2 = \text{phase origin of source 2}$$

<u>synchrony</u>:

$$\psi_1(\vec{r}_1) = \psi_2(\vec{r}_2) = \cos(0)$$

$$\Rightarrow \phi_1 = \vec{k}\cdot\vec{r}_1 = k_x X_1$$

$$\phi_2 = \vec{k}\cdot\vec{r}_2 = k_x X_2$$

Now combine these waves, to see how they contribute to the wave with vector $\vec{k}$:

$$\psi_1 + \psi_2 = \cos(\vec{k}\cdot\vec{r} - \phi_1) + \cos(\vec{k}\cdot\vec{r} - \phi_2)$$

$$= \mathrm{Re}\Big[\underbrace{\left(e^{-i\phi_1} + e^{-i\phi_2}\right)}_{\text{net amplitude}} e^{i\vec{k}\cdot\vec{r}}\Big]$$

Finally imagine point sources evenly distributed at all x within the slit, then

$$\text{net amplitude} \propto \int_0^w e^{-i\phi(x)}\,dx$$

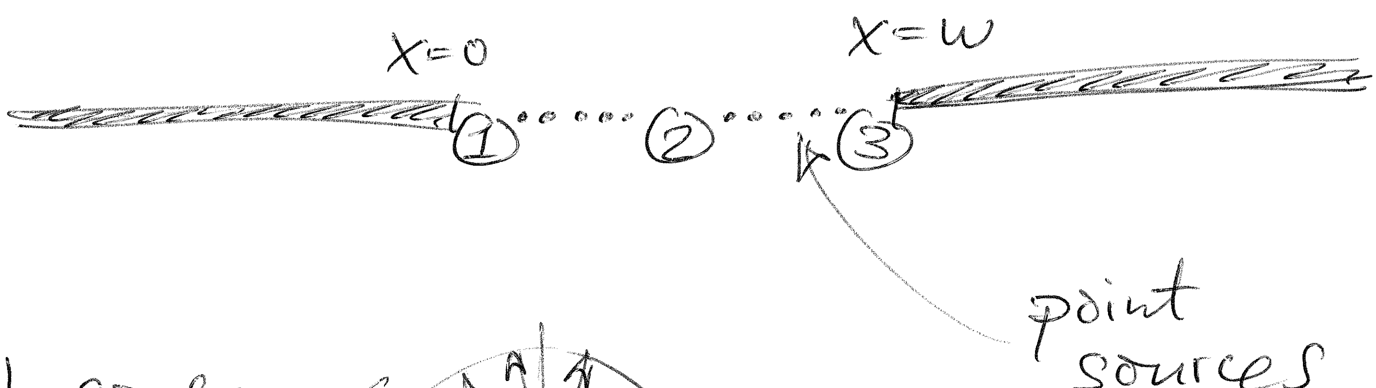
$$\phi(x) = \frac{2\pi}{\lambda} x \sin\theta = k_x x.$$

This is exactly our previous

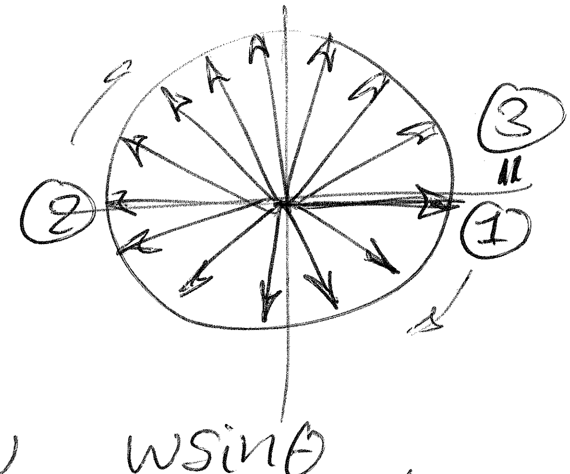
result for the diffracted wave amplitude $\hat{f}(\theta)$

Let's use the H-F principle to find the angle θ where the slit-diffraction amplitude first vanishes:

$$e^{-i\phi(x)} = e^{-i2\pi \frac{x\sin\theta}{\lambda}}$$



phasors:

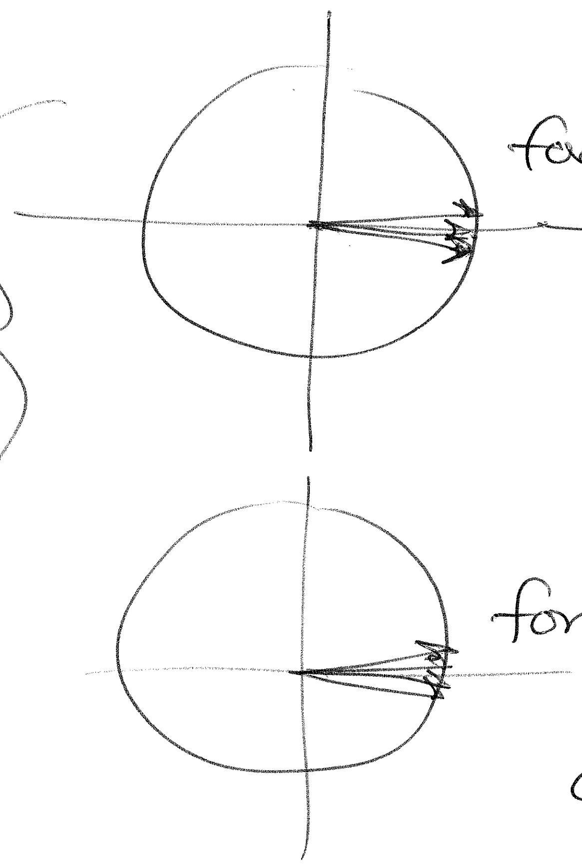


phasors wind around once to give sum $= 0$

③: $x = w$ $\quad \dfrac{w\sin\theta}{\lambda} = 1 \implies \sin\theta = \lambda/w$

Now consider two very narrow slits separated by distance d. For what θ's does the diffraction amplitude have a <u>maximum</u> magnitude ?

phasors:



for slit ①

$\phi = 0$

for max. phasor sum magnitude

for slit ②

$\phi = 2\pi N$

N = integer

$$\Rightarrow \frac{d \sin\theta}{\lambda} = N$$