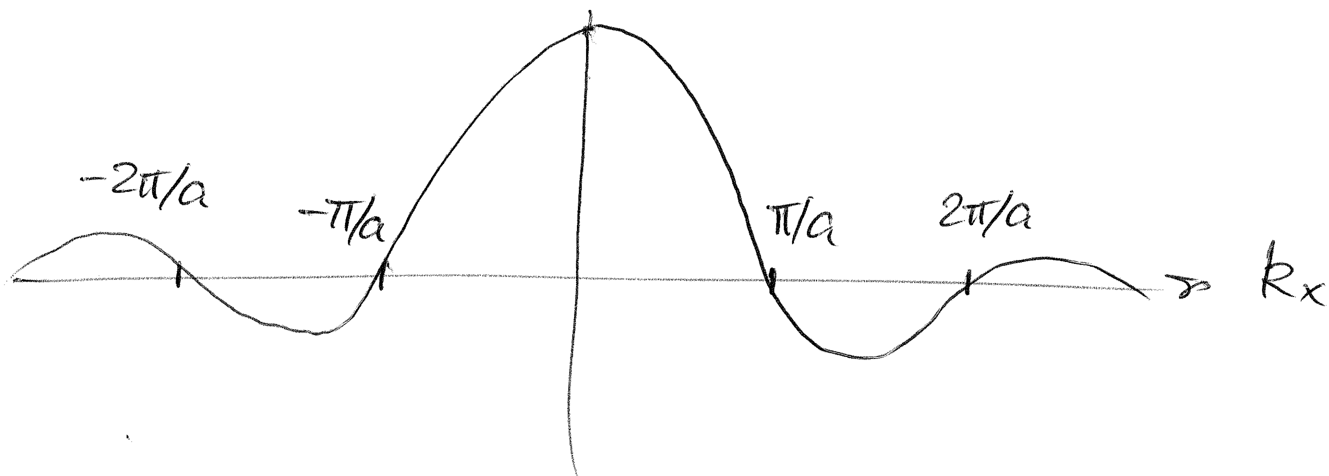
Lecture 25

$$\hat{f}(k_x) = \frac{\tilde{A}}{2\pi} \int_{-a}^{a} e^{-ik_x x}\, dx$$

$$= \frac{\tilde{A}}{\pi}\, a \left(\frac{\sin k_x a}{k_x a} \right)$$

"sinc function"



This amplitude is well-defined for any k_x — even when $|k_x| > k$!!

What does this case correspond to ?

$$k_y = \sqrt{k^2 - k_x^2} = \sqrt{-\underbrace{(k_x^2 - k^2)}_{\text{positive}}} = i \underbrace{\sqrt{k_x^2 - k^2}}_{\text{pos.}}$$

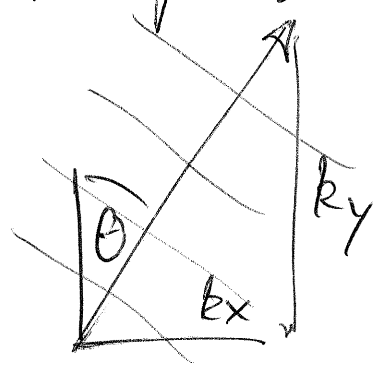
$$\varphi = \mathrm{Re}\left[\tilde{\varphi}\, e^{-i\omega t}\right]$$

$$\propto \mathrm{Re}\left[e^{i k_x x + i\left(i\sqrt{k_x^2 - k^2}\right)y - i\omega t}\right]$$

$$= \underbrace{e^{-\sqrt{k_x^2-k^2}\,y}}_{\text{decay in y-direction}} \underbrace{\cos(k_x x - \omega t)}_{\text{running wave in x-direction}}$$

So the diffracted waves with $|k_x| > k$ do <u>not</u> propagate to $y = +\infty$; they are called "evanescent" and "stick" to the screen ($y=0$). We put these aside from now on and work just with the propagating waves which have $|k_x| < k$.

For propagating waves we can define a propagation angle θ:



$$k \sin\theta = k_x$$

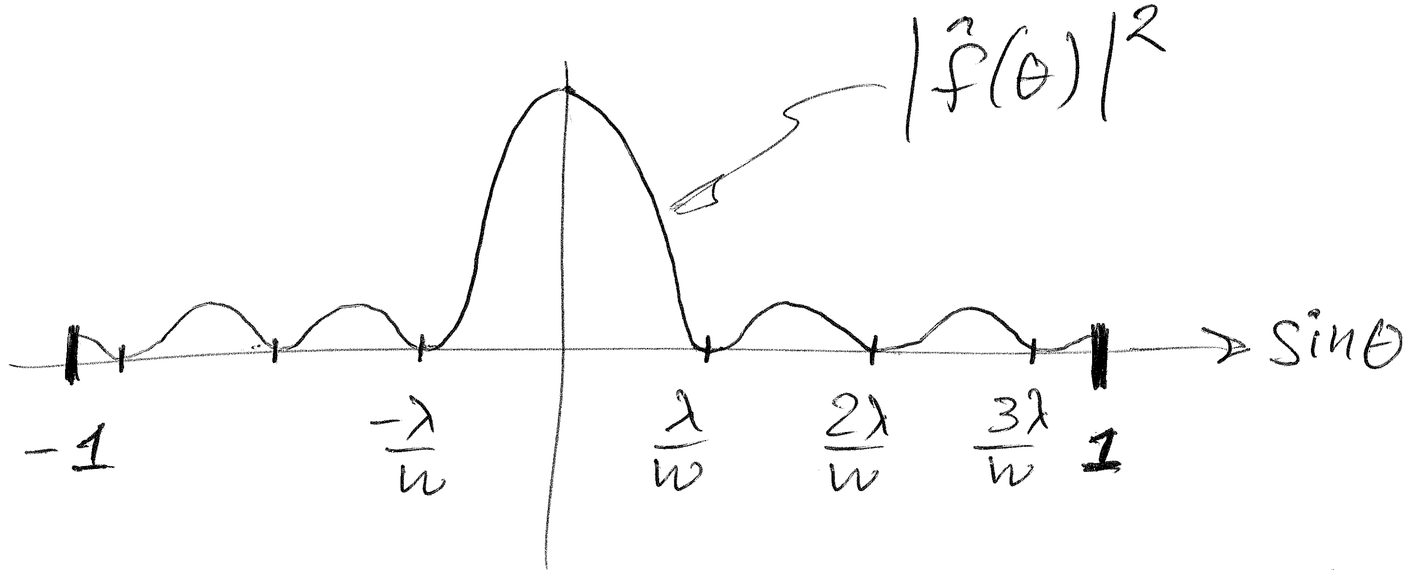
$$k_x a = k a \sin\theta = \frac{2\pi}{\lambda} a \sin\theta$$

$$= \pi \frac{W}{\lambda} \sin\theta$$

$2a = W$ = width of slit

We think of the amplitude $\hat{f}$ now as a function of θ instead of k_x (for the propagating waves):

$$\hat{f}(\theta) = \frac{\tilde{A}}{2\pi} w \underbrace{\frac{\sin\left(\pi \frac{w}{\lambda} \sin\theta\right)}{\left(\pi \frac{w}{\lambda} \sin\theta\right)}}_{\text{"sinc"}}$$

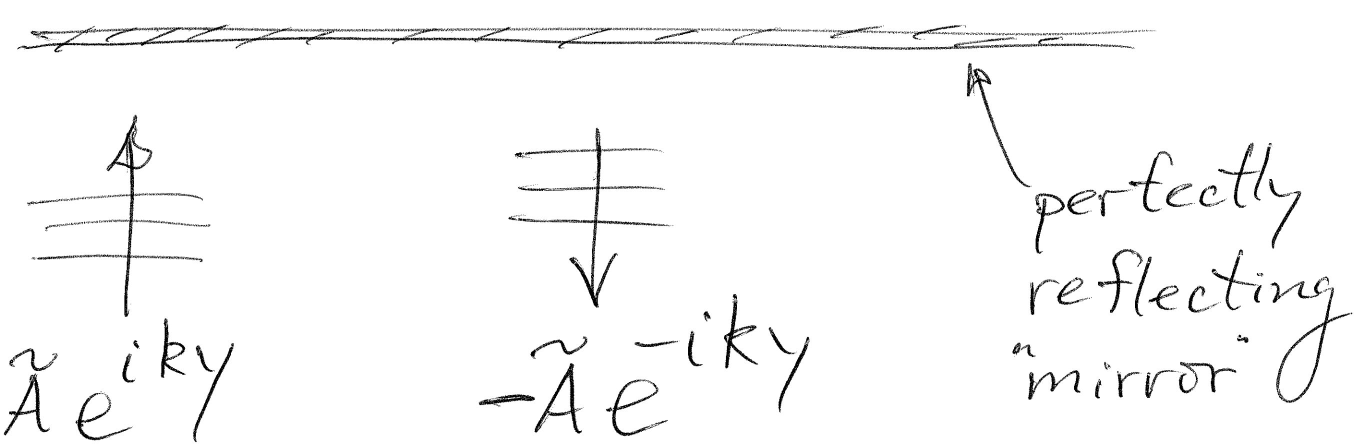


We have plotted the squared amplitude because this is

~~wh~~ proportional to (25.5)
what is usually measured — the energy or intensity of the wave. More on this later...

In the plot we chose the ratio λ/w so that $\frac{4\lambda}{w} > 1$.
Since $\sin\theta < 1$, our diffraction intensity plot will therefore have only 3+3 places where it vanishes ($\sin\theta = \pm\lambda/w, \pm 2\lambda/w, \pm\frac{3\lambda}{w}$).

We assumed the screen was perfectly absorbing to the incident wave. When it is reflecting there is also a reflected wave. On a perfectly reflecting surface the net wave amplitude must be <u>zero</u> at all times.



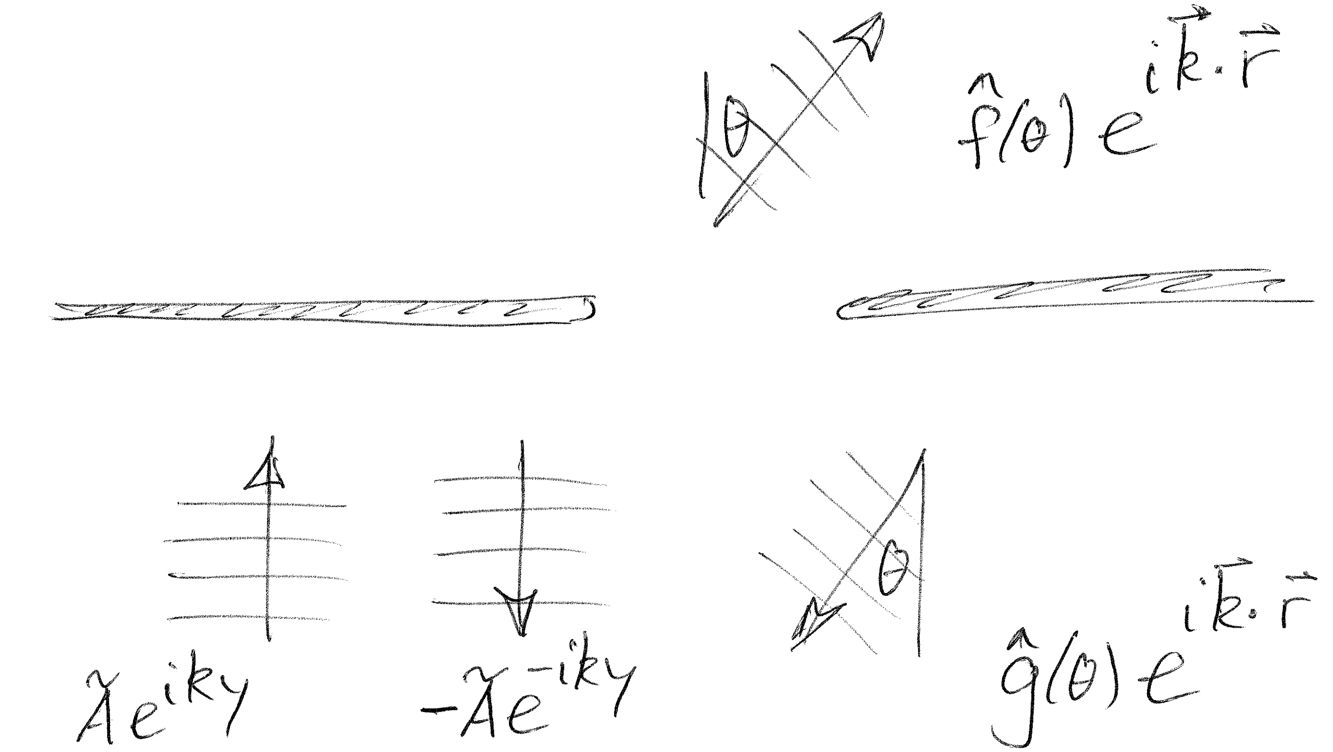
$\tilde{A}$ = real, when mirror at $y = 0$

$$\psi = \mathrm{Re}\left[\tilde{A} e^{iky - i\omega t} - \tilde{A} e^{-iky - i\omega t} \right]$$

$$\psi(y=0) = \tilde{A}\left[\cos(-\omega t) - \cos(-\omega t) \right]$$

$$= 0$$

When the screen of our slit is made of a reflecting material, we need all of the waves shown below:



The diffracted-reflected waves with amplitude $\hat{g}(\theta)$ come about because the reflected wave $-\tilde{A} e^{-iky}$ must be cancelled at

25.8

the slit opening,
since there is no reflecting
screen there.