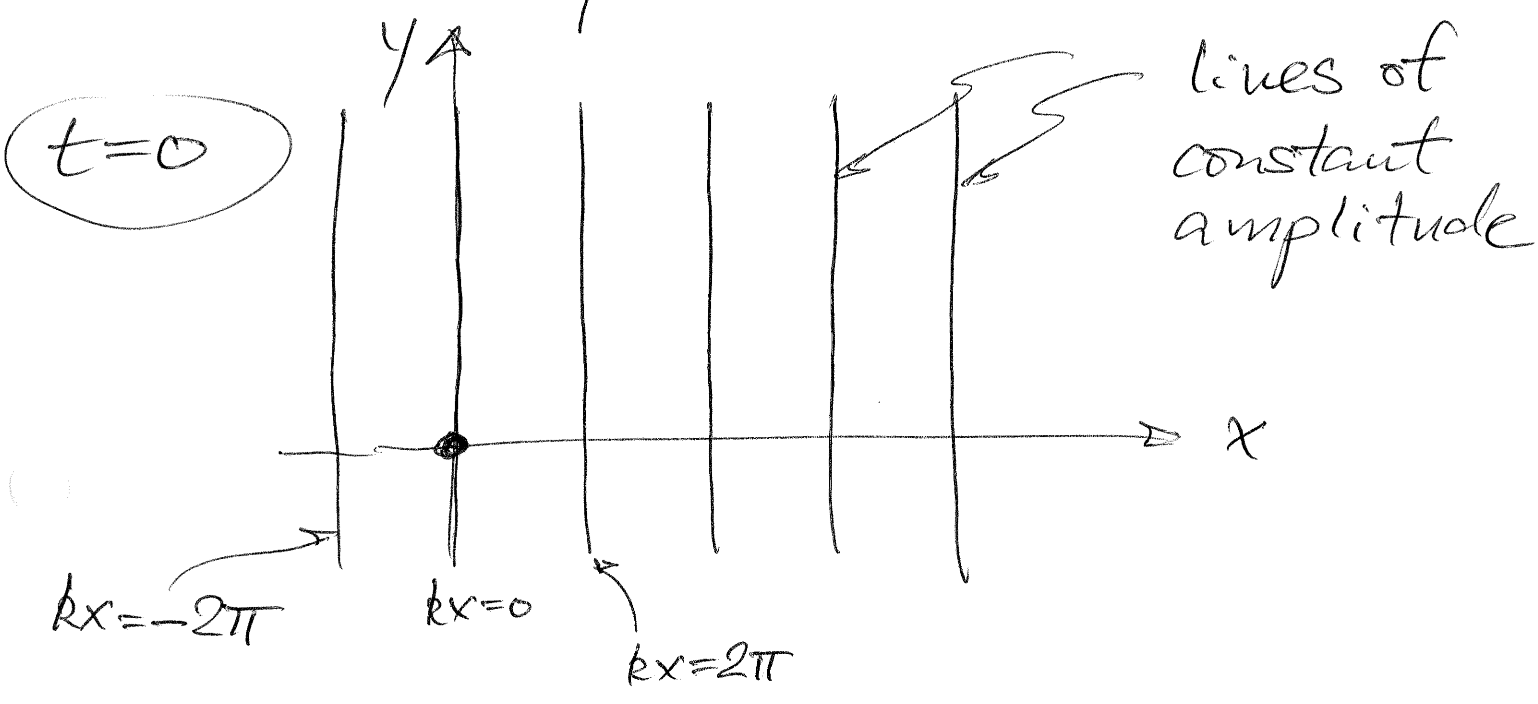
Lecture 24

Our waves so far have had oscillations in only one spatial dimension (x) — their amplitudes were constant in the other dimensions (y & z). For example,

running wave in <u>+x</u> <u>direction</u>:

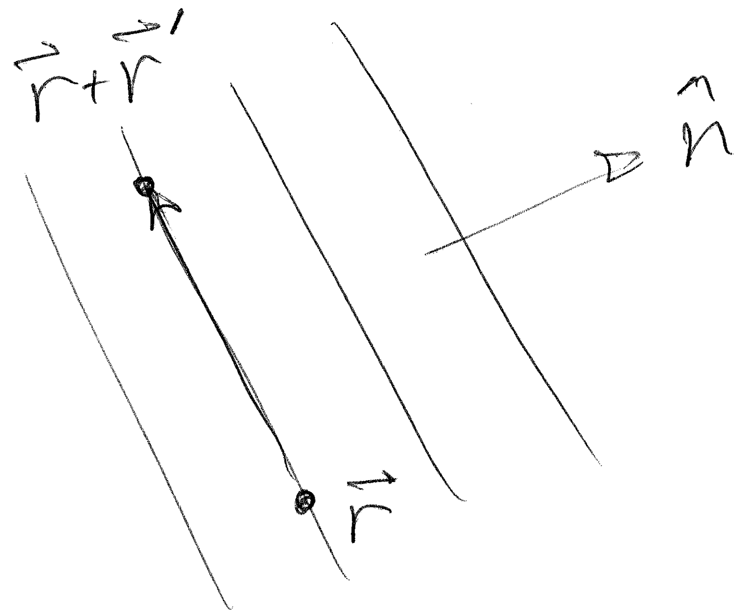
$$\psi(x,y,t) = A\cos(kx - \omega t + \phi)$$

cartoon representation:



running wave in $\hat{n}$-direction:
($\hat{n}$ = unit vector)

$$\psi(\vec{r},t) = A\cos(k\,\hat{n}\cdot\vec{r} - \omega t + \phi)$$



$\vec{r}' \perp \hat{n}$

check: $\psi(\vec{r}+\vec{r}',t) = \psi(\vec{r},t)$

$$\psi(\vec{r}+\lambda\hat{n},t) = \psi(\vec{r},t)$$

$$k\hat{n}\cdot(\vec{r}+\lambda\hat{n}) = k\hat{n}\cdot\vec{r} + \lambda k$$
$$= k\hat{n}\cdot\vec{r} + 2\pi$$

define "wave-vector":

$$\vec{k} = k\hat{n} \quad , \quad k_x^2 + k_y^2 = k^2$$

$$\psi(x,y,t) = A\cos(k_x x + k_y y - \omega t + \phi)$$

---

complex exponential notation

$$\psi(\vec{r},t) = \mathrm{Re}\Big[\underbrace{Ae^{i\phi}}\, e^{i\vec{k}\cdot\vec{r}}\, e^{-i\omega t}\Big]$$

$\tilde{A}$ = complex amplitude

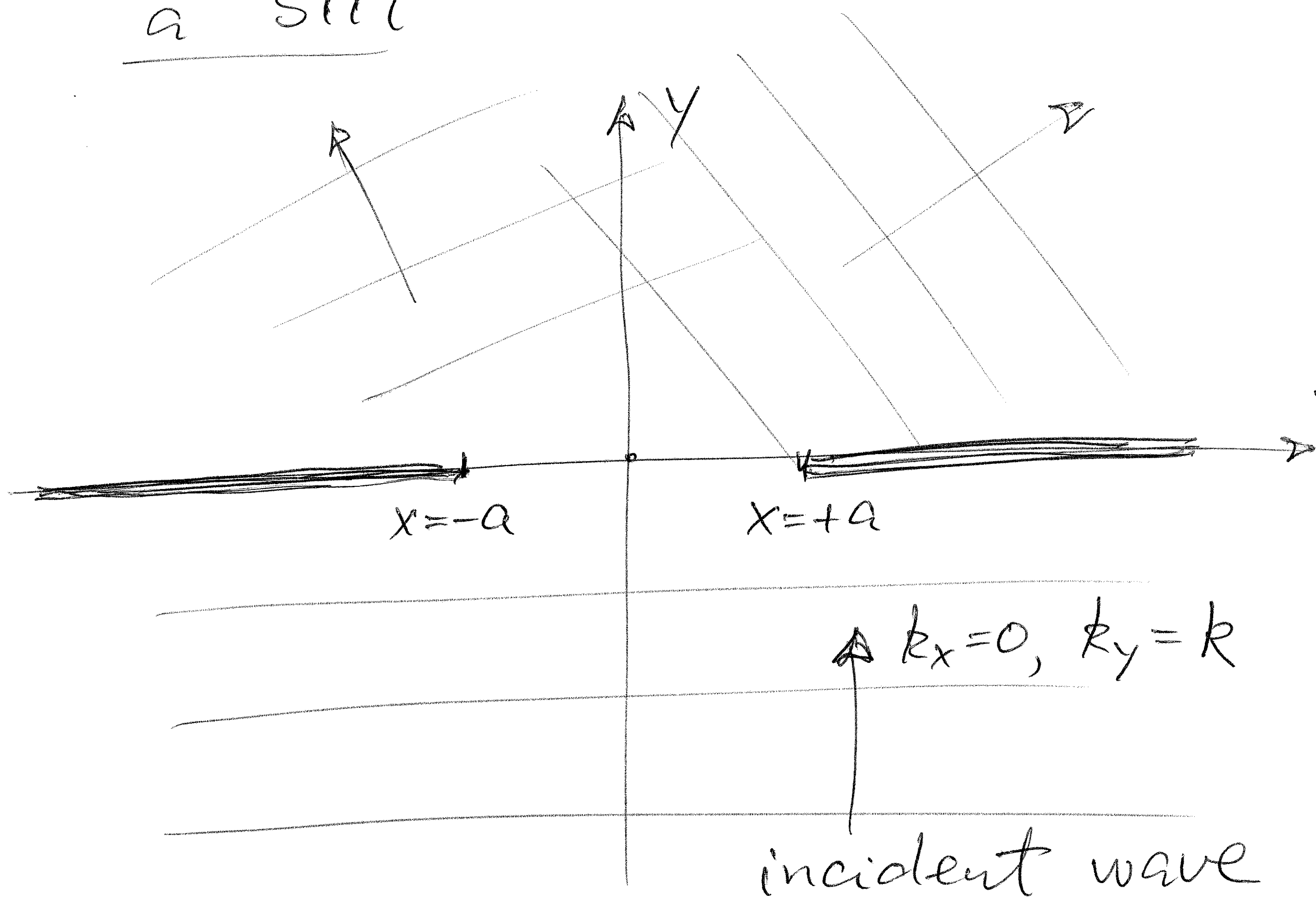
In much of what we do in the next several lectures the waves have the same $e^{-i\omega t}$ time dependence. We therefore use the compact notation

$$\tilde{\Phi} = \tilde{A}\, e^{i\vec{k}\cdot\vec{r}}$$

for a "plane wave propagating in $\vec{k}$-direction with amplitude (magnitude & phase) $\tilde{A}$." The understanding is that we recover the actual wave at time t by

$$\psi(\vec{r},t) = Re\left[\tilde{\Phi}(r)\, e^{-i\omega t}\right]$$

diffraction through a slit



idea: superpose plane waves in $y>0$ (above slit) to match the incident wave amplitude in $y<0$ (from below slit)

$y<0$: $\tilde{\Psi} = \tilde{A} e^{iky}$

$y>0$: $\tilde{\Psi} = \int \hat{f}(k_x) e^{ik_x x + ik_y y} dk_x$

$\hat{f}(k_x)$ = amplitude of wave in superposition whose x-component of $\vec{k}$ equals k_x

k_y is determined by k_x:

$k_x^2 + k_y^2 = k^2 \leftarrow$ set by ω

$k_y = \sqrt{k^2 - k_x^2}$

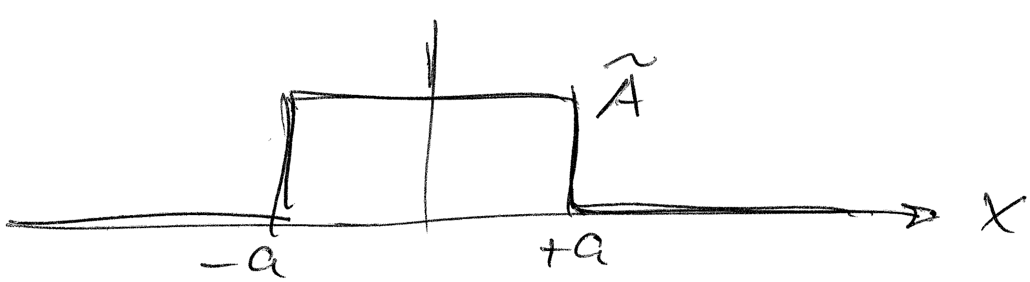
take positive root, otherwise diffracted waves running ~~to~~

toward slit, not away from slit as they should (there is no source of waves at $y = +\infty$).

The two wave expressions ($y>0$ & $y<0$) must match at $y=0$:

incident:

$$\tilde{\Psi}(x,0) = f(x) = \begin{cases} \tilde{A} & -a < x < +a \\ 0 & \text{otherwise} \end{cases}$$



diffracted:

$$\tilde{\Psi}(x,0) = \int \hat{f}(k_x)\, e^{ik_x x}\, dk_x$$

matching condition:

$$f(x) = \int \hat{f}(k_x)\, e^{ik_x x}\, dk_x$$

known (set by slit)

to be determined

This equation states that $f(x)$ is the Fourier transform of $\hat{f}(k_x)$. Since we want to know $\hat{f}(k_x)$, we can use the inverse transform:

$$\hat{f}(k_x) = \int \frac{dx}{2\pi}\, f(x)\, e^{-ik_x x}$$