Conceptual physics exercise:
series vs. parallel lumped capacitors of Q and V, which are shared and which are summed?

Here is a schematic drawing showing the construction of the 1F "super capacitor":

\[
2H^+ + SO_4^{2-} \rightarrow 0
\]
This is actually like two capacitors in series:

\[
\begin{array}{c}
\uparrow & \Downarrow & \uparrow \\
+ & 0 & 0 \\
+ & 0 & 0 \\
+ & 0 & 0 \\
\end{array}
\]

\[C_{series} = \left( \frac{1}{C_1} + \frac{1}{C_2} \right)^{-1}\]

The ions coat the surface of the carbon electrodes so the separation of the "plates" is only on the order of a molecular diameter!\[d = 10^{9} \mu m = 1 \text{ nm}\]

We will model \(C_1\) and \(C_2\) as parallel plate capacitors of area \(A\).
\[ C_1 = C_2 = \varepsilon_0 \frac{A}{d}, \quad C_{\text{series}} = \frac{C_i}{2} \]

\[ \frac{C_i}{A} = \frac{\varepsilon_0}{d} \sim \frac{10^{-11} \text{ F/m}}{10^{-9} \text{ m}} = 10^{-2} \text{ F/m}^2 \]

\[ C_{\text{series}} = 1 \text{ F} \Rightarrow C_i = 2 \text{ F} \]

\[ \Rightarrow A = 200 \text{ m}^2 ! \]

It's possible to fit 200 m² area into a space that you can hold in your hand by compacting into a solid small particles of carbon:

small carbon particles with enormous interstitial area possible to get \( \frac{1000 \text{ m}^2}{\text{gm}} \)

\( \Rightarrow \frac{1}{5} \text{ gm carbon/electrode} \)
In the next few lectures we will be comparing the electric field, at a particular space-time event, as observed in different inertial reference frames. As a 3-vector, we already know how $\mathbf{E}$ and $\mathbf{E}'$ are related when one frame is simply rotated with respect to the other. What about frames in relative (uniform) motion? If $\mathbf{E}$ could be generalized as a 4-vector, then we would know what to do. However, that turns out not to be the case. The transformation rule, relating $\mathbf{E}$ and $\mathbf{E}'$ in relatively moving frames, is actually quite simple.
and we will arrive at it by applying very basic principles. These include:

- symmetry
- special theory of relativity (as it relates to distances)
- invariance of charge
- field principle
  (E exists as a local entity, independent of its sources.)

We'll start by comparing the electric fields produced by very symmetric distributions of charge — infinite planar sheets — when the motion is parallel to the planes. A uniform sheet of charge will still be
a uniform sheet of charge in the moving frame. One thing that will be different is the density of charge—a consequence of Lorentz contraction. However, according to the "invariance of charge", the quantity of charge enclosed by a Gaussian surface is unchanged by the state of motion of the charge. Our sheets of charge will be rendered in cross section like this:
We have two parallel sheets \( \Sigma \) in the \( x-z \) plane with charge density \( \pm \sigma \) in the rest frame of the charge. The electric field will be purely in the \( y \)-direction with magnitude \( \frac{\sigma}{\epsilon_0} \).

What electric field is seen by an observer moving with speed \( v \) in the \( x \)-direction? The vanishing of \( \vec{E} \) above and below the sheets and its perpendicular direction (relative to the sheets) will be unchanged by the motion of the charges in this frame. It's true that the state of motion of the charges undermines the right-left symmetry of the \( x \)-axis, so
the field produced by just the +σ sheet might look like:

\[ \text{charges moving to left} \]

The field of the -σ sheet would then be

Superposing these gives a perpendicular field that is nonzero
only between the sheets—exactly as in the rest frame. So the only thing that might be different is the magnitude of \( \vec{E} \). We'll determine this using Gauss's law, which we know (from experiments with rapidly moving elementary charges) is valid even when the charges enclosed are moving. Now a Gaussian surface at rest in the primed (moving) frame will enclose an amount of charge that we can calculate using the rules of special relativity. The spacing of charges will be contracted.
in the $x'$ direction and unchanged in the $z'$ direction, perpendicular to the motion. The contraction in $x'$ is by the factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \geq 1,$$

and so the charge density in the moving frame is enhanced as

$$\rho' = \gamma \rho.$$

Here is a quick review of the Lorentz contraction effect. We will focus on two fixed markers in one sheet at different $x$ in the rest frame; these will have slanted world lines in the moving frame.
Now we do some similar triangle geometry on the triangle highlighted above:
\[ d'' = \text{(distance between markers in primed frame, when they are simultaneous in that frame)} \]

\[ d' = \frac{c^2}{\sqrt{\gamma}} \gamma - \sqrt{\gamma} = \frac{c^2}{\sqrt{\gamma}} \gamma (1 - \frac{v^2}{c^2}) \]

\[ d = \text{(distance between markers in un-primed frame)} \]

\[ = (\text{proper distance between events A and B}) \]

\[ = \sqrt{\left(\frac{c^2}{\sqrt{\gamma}} \gamma \right)^2 - (c \gamma)^2} \]

\[ = \sqrt{\left(\frac{c^2}{\sqrt{\gamma}} \gamma \right)^2 - 1} \cdot c \gamma \]

\[ = \sqrt{1 - \frac{v^2}{c^2}} \cdot \frac{c^2}{\sqrt{\gamma}} \gamma \]
\[
\frac{d''}{d} = \frac{1 - \nu^2 c^2}{\sqrt{1 - \nu^2 c^2}} = \sqrt{1 - \nu^2 c^2} = \frac{1}{f}
\]