Lecture 18

from previous lectures:

$$S(x,t) = \int_{-\infty}^{\infty} dk\ \hat{S}(k,t)\, e^{ikx}$$

(relation between function and its Fourier trans.)

$$S_R(x,t) = \int_{0}^{\infty} h(k) \cos(kx - \omega(k)t + \phi(k))\, dk$$

(most general right-moving wave for dispersion relation $\omega(k)$)

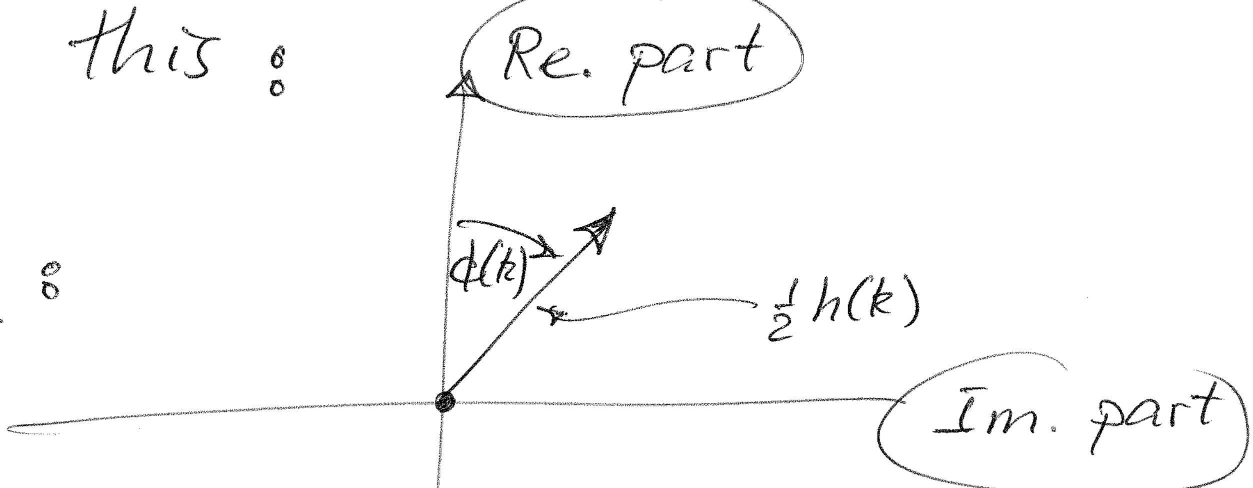
We rewrote 2^{nd} equation so it looked like 1^{st} equation and found:

$$\hat{S}_R(k,t) = \begin{cases} \frac{1}{2} h(k)\, e^{i\phi(k) - i\omega(k)t}, & \underline{\underline{k>0}} \\ \frac{1}{2} h(-k)\, e^{-i\phi(-k) + i\omega(-k)t}, & \underline{\underline{k<0}} \end{cases} \tag{18.2}$$

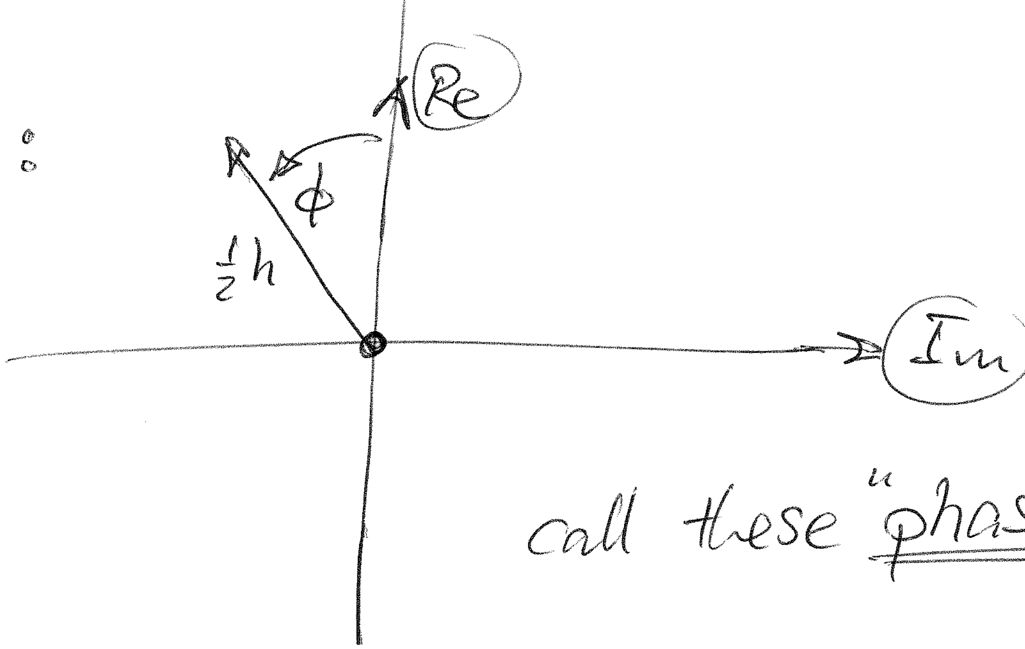
$\uparrow$ right-moving

To better visualize structure of $\hat{S}_R(k,t)$ we plot complex numbers like this:

$\underline{\underline{k>0}}$:

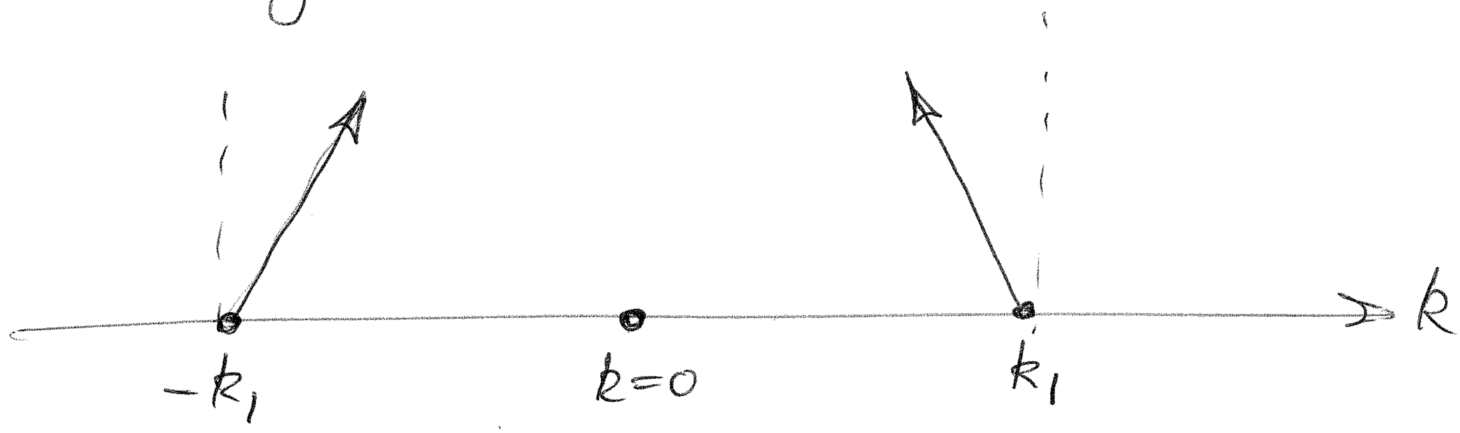


$\underline{\underline{k<0}}$:



call these "<u>phasors</u>"

Plant phasors for different k along k-axis:



These diagrams always have a mirror symmetry about $k=0$. As time advances, phasors for $k>0$ rotate as

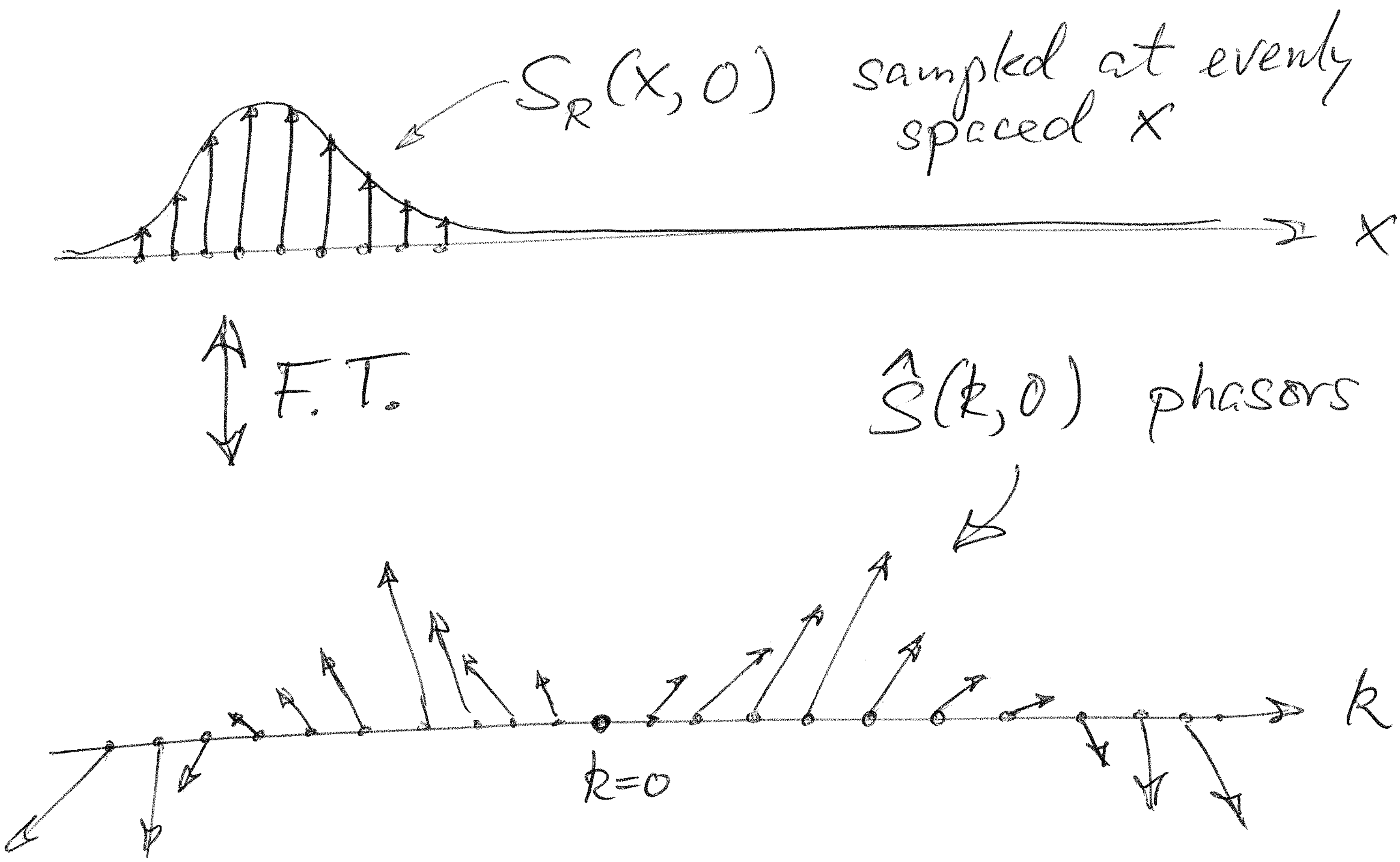
$$e^{-i\omega(k)t}$$ counter-clockwise

for $k<0$ as

$$e^{+i\omega(-k)t}$$ clockwise

This maintains the mirror symmetry

wave dynamics in "Fourier space" is simpler than in real space. Start with $t=0$:



F.T. = Fourier transform
(can be very <u>efficiently</u> computed on evenly sampled functions using FFT algorithm)

"fast Fourier transform"

(18.5)

At $t>0$ the function $S_R(x,t)$ may have a very complex dynamics (see demo) while $\hat{S}_R(k,t)$ changes very simply and predictably: the phasors rotate at a rate and with a sense that depends on where they are on the k-axis.