Lecture 16

- wave-packet demo
- dispersion applied to musical performance (Cornell Alma Mater)
- surface-wave dispersion demo

Fourier ~~trans~~ transformation between two, equivalent representations of a function:

$$S(x,t) \longleftrightarrow \hat{S}(k,t)$$

$\hat{S}$ = "Fourier transform of S"

x, k = transform variables

(treat t as a constant parameter in what we do in next few lectures)

Definition of Fourier transform:

$$\hat{S}(k,t) = \int_{-\infty}^{\infty} \frac{dx}{2\pi}\, S(x,t)\, e^{-ikx} \qquad (A)$$

Can recover $S(x,t)$ from $\hat{S}(k,t)$ like this:

$$S(x,t) = \int_{-\infty}^{\infty} dk'\, \hat{S}(k',t)\, e^{ik'x} \qquad (B)$$

Need to check that this is correct

Substitute claimed $S(x,t)$ (16.3)

from (B) into (A):

$$\hat{S}(k,t) \overset{?}{=} \underbrace{\int_{-\infty}^{\infty} \frac{dx}{2\pi} \left[\int_{-\infty}^{\infty} dk'\right.}_{\text{2D integral: switch order}} \underbrace{\left. \hat{S}(k',t)\, e^{ik'x} \right] e^{-ikx}}_{\text{integrand: } e^{i(k'-k)x}\ \hat{S}(k',t)}$$

$$\hat{S}(k,t) \overset{?}{=} \int_{-\infty}^{\infty} dk' \underbrace{\int_{-\infty}^{\infty} \frac{dx}{2\pi}\, e^{i(k'-k)x}}_{\text{do this first!}} \ \hat{S}(k',t)$$

$\uparrow$ no x