Lecture 15

A "resistor" is one of the basic components of electrical circuits. It has the property that current \( I = (V_a - V_b) / R \) flows between its two terminals when these have potential difference \( V_a - V_b \):

![Diagram of resistor with current flow](image)

The direction of positive current flow indicated above...
in the circuit diagram implies $V_a > V_b$. This is consistent with $\vec{E} = -\nabla \Phi$, as $\vec{E}$ is always in the same direction as $I$ by

$$\vec{J} = \frac{1}{\rho} \vec{E}.$$ 

Most circuits are designed with resistors in mind that have a constant value of $R$. However, as we have seen, physical parameters such as temperature can change $R$. A charge (e.g. conduction electron) moving through a
A resistor loses energy to frictional forces. The rate of energy loss, or power, for all the free charge that moves between the terminals during time $\Delta t$ is

$$P = \frac{\Delta Q \cdot V_a - \Delta Q \cdot V_b}{\Delta t}$$

$$= \frac{\Delta Q}{\Delta t} V = IV$$

Using Ohm's Law this can be also written using just $I$ or just $V$:

$$P = IV = I^2R = V^2/R$$
Even though both $I$ and $V$ involve electrical units, the combination $IV$ is just the mechanical unit of power: Watt

$$Amp \times Volt = \frac{C}{S} \times \frac{J}{C} = \frac{J}{S} = W$$

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Energy/power is introduced in a circuit by batteries. The origin of the energy is chemical: the formation of lower energy molecules in processes that involve the transfer of charge. We will explain the battery mechanism using the
example of the lead-acid storage battery:

Sulfuric acid:

\[ \text{H}_2\text{SO}_4 \rightarrow 2\text{H}^+ + \text{SO}_4^{2-} \]

Ions in solution

Chemist's view:

Chemical reactions take place at the surface of the lead-
oxide electrode (A) and the surface of the lead electrode (B):

(A): \[ 2e^- + PbO_2 + 4H^+ + SO_4^{2-} \]
\[ \rightarrow \]
\[ PbSO_4 + 2H_2O \quad + 3.37 \text{ eV} \]
\[ \text{low energy molecules} \]

(B): \[ SO_4^{2-} + Pb \rightarrow PbSO_4 + 2e^- \]
\[ \text{low energy} \quad + 0.71 \text{ eV} \]

Notice that whenever a pair of these reactions takes place, charge -2e flows directly
from the Pb electrode to the PbO₂ electrode (via the circuit that the battery is connected to) while charge +2e flows internally (through the solution) between the same pair of electrodes.

**Physicist's View**

- External: +2e
- Internal: +2e

Net chemical energy gain when charge +2e moves around loop:

\[ 3.37 \text{ eV} + 0.71 \text{ eV} = 4.08 \text{ eV} \]
The "electromotive force" (emf) of the battery is defined as the energy gain per unit charge that moves through the battery:

\[ \text{emf} = E = \frac{4.08 \text{ eV}}{2e} = 2.04 \text{ Volts} \]

This energy output of the battery is dissipated by both the external and internal parts of the circuit.

\[ I \rightarrow R(\text{external resistance}) \rightarrow \bullet \rightarrow r(\text{internal resistance}) \]
\[ E = IR + Ir \]

We should contrast \( E \) with the potential "seen" by the external circuit, the "terminal voltage" \( V \):

\[ V = IR = \left( \frac{E}{R+r} \right)R \]

\[ = \left( \frac{R}{R+r} \right)E < V \]

Normally \( r \ll R \) and \( V \approx E \), but when a battery "runs low" it's usually because \( r \) is large.