Lecture 13

surface waves:

$$\omega(k) = \sqrt{4\, k\, f(k)/\rho}$$

HW4: $f(k) = \frac{1}{4}\rho g + \frac{1}{4}\sigma k^2$

grav. potential energy

surface energy

$$\boxed{\omega(k) = \sqrt{gk + \frac{\sigma k^3}{\rho}}}$$

$k \to 0,\ \lambda \to \infty\ :\ \omega \simeq \sqrt{gk}$

"gravity waves"

$k \to \infty,\ \lambda \to 0\ :\ \omega \simeq \sqrt{\frac{\sigma k^3}{\rho}}$

"capillary waves"

Compare with simple wave equation

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

$\psi = H\cos(kx - \omega t)$

$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$

$-\omega^2 = v^2(-k^2)$

$\frac{\partial^2 \psi}{\partial x^2} = -k^2 \psi \quad \Rightarrow \omega(k) = v k$

<u>linear</u> dispersion

Simple wave equation sol.:

$$\psi(x,t) = f(x-vt)$$

arbitrary

waves, pulses, etc. all move with speed v. Need to work harder when $\omega(k)$ is <u>not</u> linear function!

HW 5: starting with <u>standing-wave</u> solutions of surface wave equations, such as

$$S(x,t) = h(t)\cos kx$$

SHO: $h(t) = H\cos\omega_k t$,

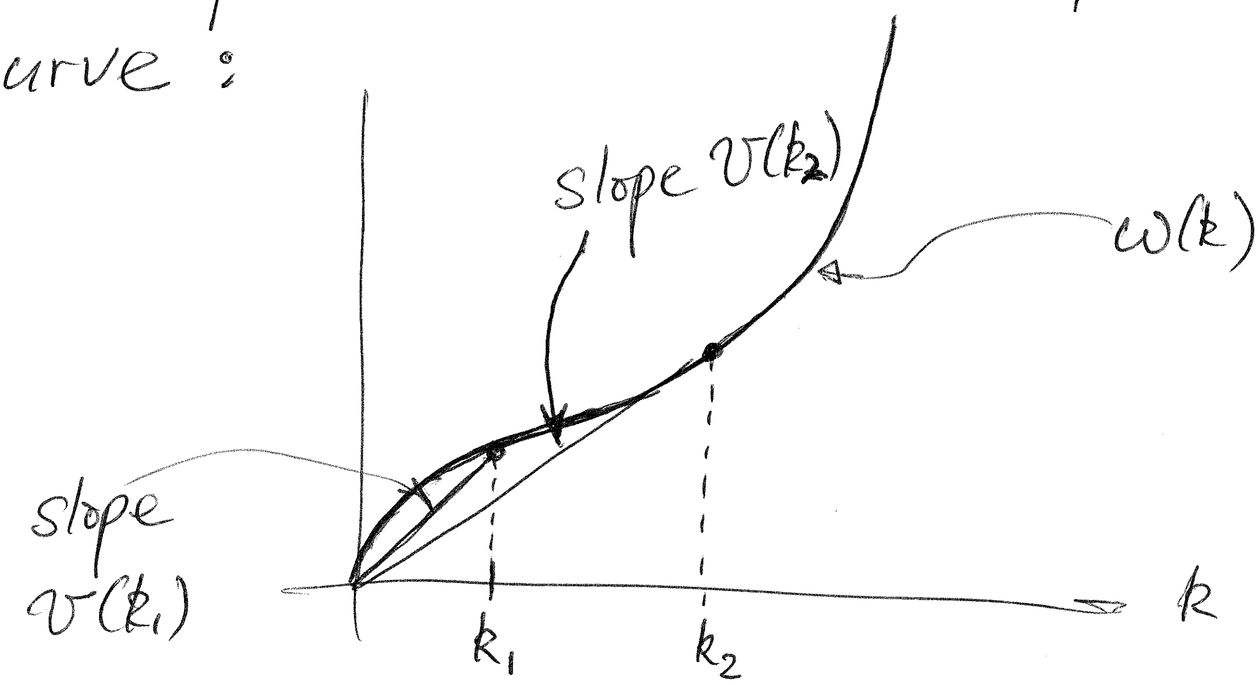
construct the <u>running-wave</u>

$$S(x,t) = H\cos(kx - \omega_k t)$$

as a suitable superposition. This running wave moves to the right with speed

$$v(k) = \omega_k / k = \frac{\omega(k)}{k}$$

where $\omega(k)$ is the dispersion relation. Graphically, $v(k)$ is the slope of chord to dispersion curve:



The speed defined by

$v(k) = \frac{\omega(k)}{k}$ only applies to <u>infinite</u> sinusoidal waves with a single k. For finding the propagation behavior of <u>finite</u> signals we need something more general. We will consider a special class of waves called "wave packets" formed by superposing sinusoidal waves with k in a small range about some central k_0: