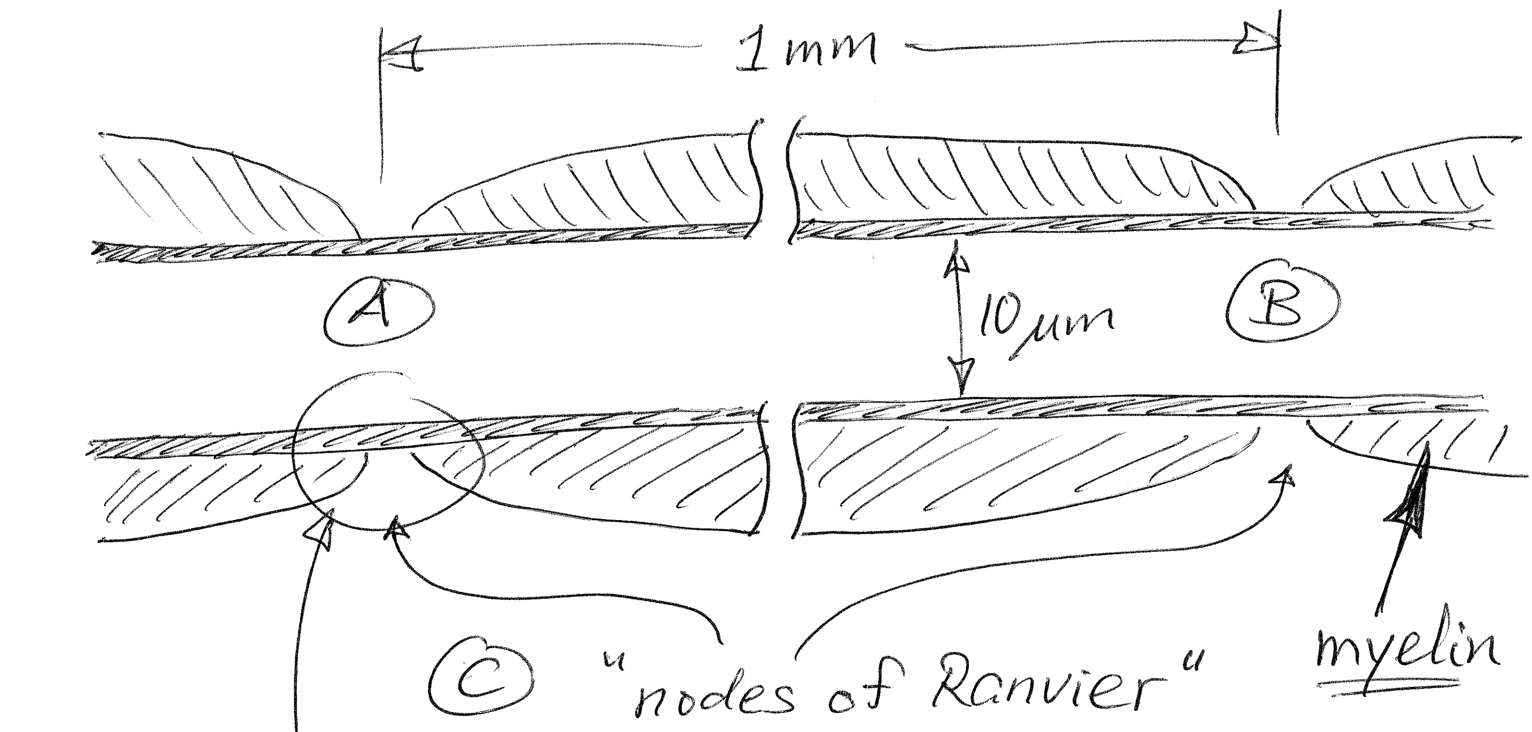
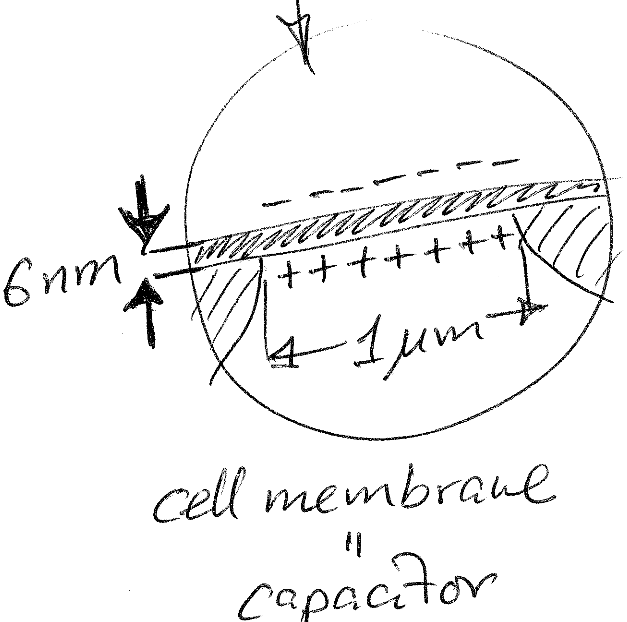
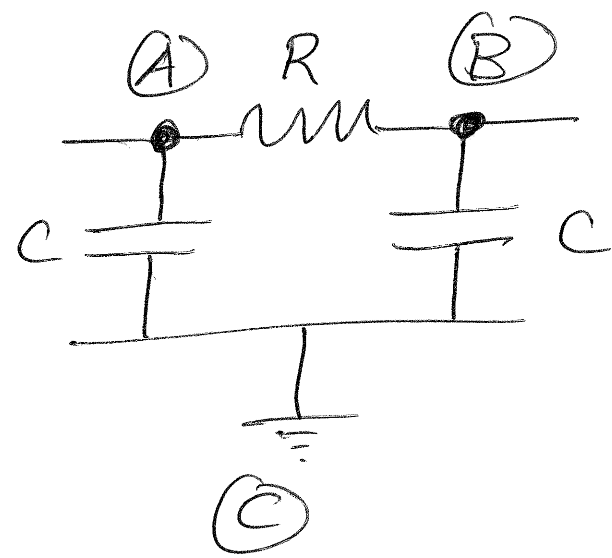
Lecture 12

myelinated nerve fiber for HW 4 :

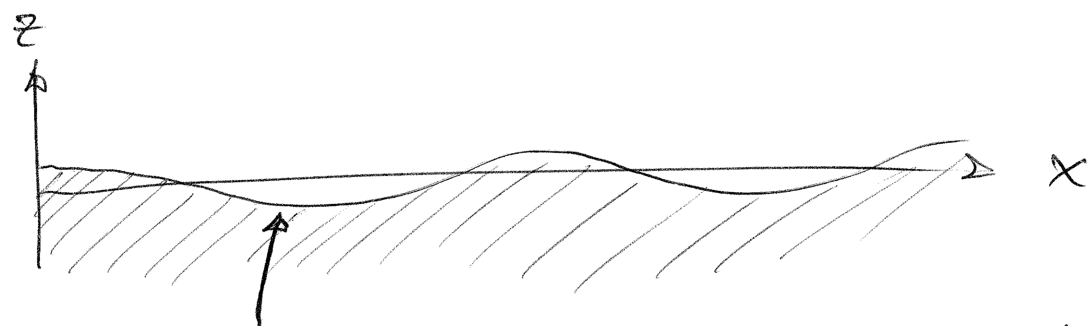






surface-wave oscillations

1) surface/flow boundary conditions
2) kinetic energy in flow
3) stored energy of wave
4) surface-wave harmonic oscillator



$s(x,t)$ = vertical displacement of fluid surface

$$s(x,t) = h(t)\cos kx$$

$h(t)$ = "mode amplitude"

vertical surface velocity =

$$\dot{s}(x,t) = \dot{h}(t)\cos kx$$

boundary condition:

$$\dot{s}(x,t) \underset{\substack{\uparrow \\ \text{small } h}}{\cong} V_z(x, z=0) \underset{\substack{\uparrow \\ \text{see lec. 11} \\ \text{notes}}}{=} kA\cos kx$$

$$\Longrightarrow \quad \dot{h}(t) = kA$$

$$V_x = -\dot{h}\sin kx\, e^{kz}$$

$$V_z = \dot{h}\cos kx\, e^{kz}$$

flow speed:

$$v^2 = V_x^2 + V_z^2 = \dot{h}^2 e^{2kz}$$

only depth dependent

Kinetic energy in a volume of horizontal area A and unspecified depth (not relevant because $v^2 \propto e^{2kz}$ decays rapidly with depth)

$$K = \int \tfrac{1}{2}\rho v^2 \, dx\, dy\, dz$$

$$= \tfrac{1}{2}\rho \underbrace{\left(\int dx\, dy\right)}_{A} \underbrace{\left(\int_{-\infty}^{0} dz\, \dot{h}^2 e^{2kz}\right)}_{\dot{h}^2/2k}$$

$$K = \frac{\rho A}{4k}\dot{h}^2$$

$K \propto \frac{1}{k} \propto \lambda$ = thickness (in depth) of flow region stored energy:

$$U = f(k)A \quad h^2$$

$\uparrow$ HW problem

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total energy:

$$E = K + U = \frac{1}{2}\left(\frac{\rho A}{2k}\right)\dot{h}^2 + \frac{1}{2}\left(2f(k)A\right)h^2$$

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compare with SHO:

$$E = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}Kx^2$$

$$\omega = \sqrt{\frac{K}{m}}$$

$$\omega_{wave} = \sqrt{\frac{2f(k)A}{\frac{\rho A}{2k}}} = \sqrt{4k\,f(k)/\rho}$$