Lecture 11

fluids : have preferred density ρ

"incompressible flow" : $\rho = \text{const.}$

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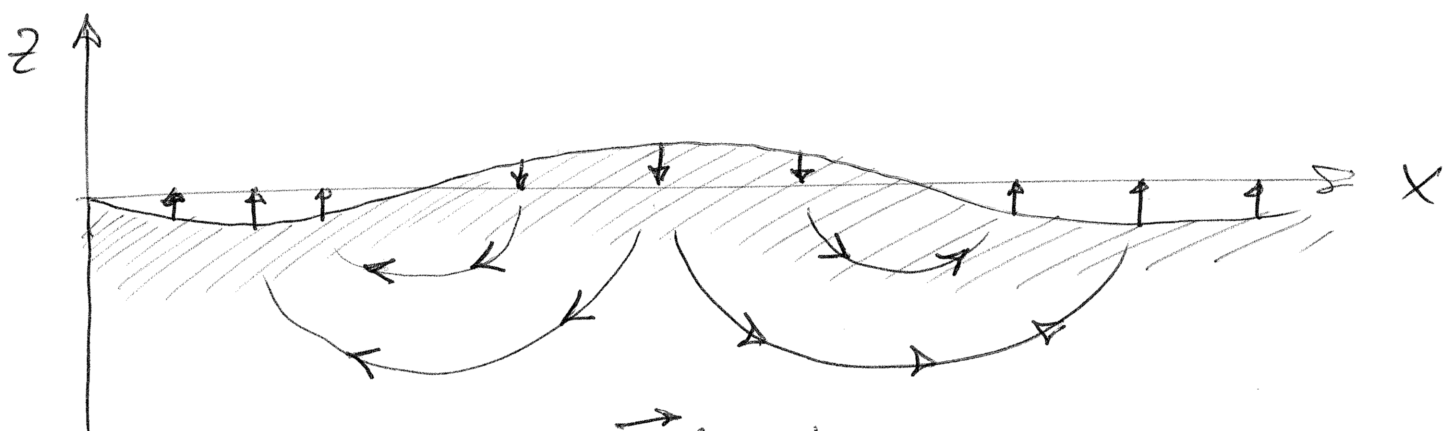
waves on surface of fluid



$\vec{V}(x, z)$

motion of surface induces flow below the surface

$\vec{V}(x,z)$ = vector field of flow

mathematical properties of flow field:

$\vec{\nabla}\cdot\vec{V}=0 \leftarrow$ incompressible flow

$\vec{\nabla}\times\vec{V}=0 \leftarrow$ zero local-angular momentum

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$\vec{V}$ is mathematically equivalent to electrostatic electric field in a region with no charge

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$$\vec{\nabla}\times\vec{V}=0 \implies \vec{V}=\vec{\nabla}\phi$$

ϕ = "flow potential"

$$\vec{\nabla}\cdot\vec{V}=0 \implies \vec{\nabla}\cdot\vec{\nabla}\phi=\nabla^2\phi=0$$

flow potential satisfies
Laplace equation

flow potential for fluid under a surface wave (will check boundary condition at surface later)

$$\phi = A \cos kx \, e^{kz}$$

$k = 2\pi/\lambda$, λ = wavelength of surface wave

check Laplace equation:

$$\frac{\partial^2 \phi}{\partial x^2} = -k^2 \phi \,, \quad \frac{\partial^2 \phi}{\partial z^2} = k^2 \phi$$

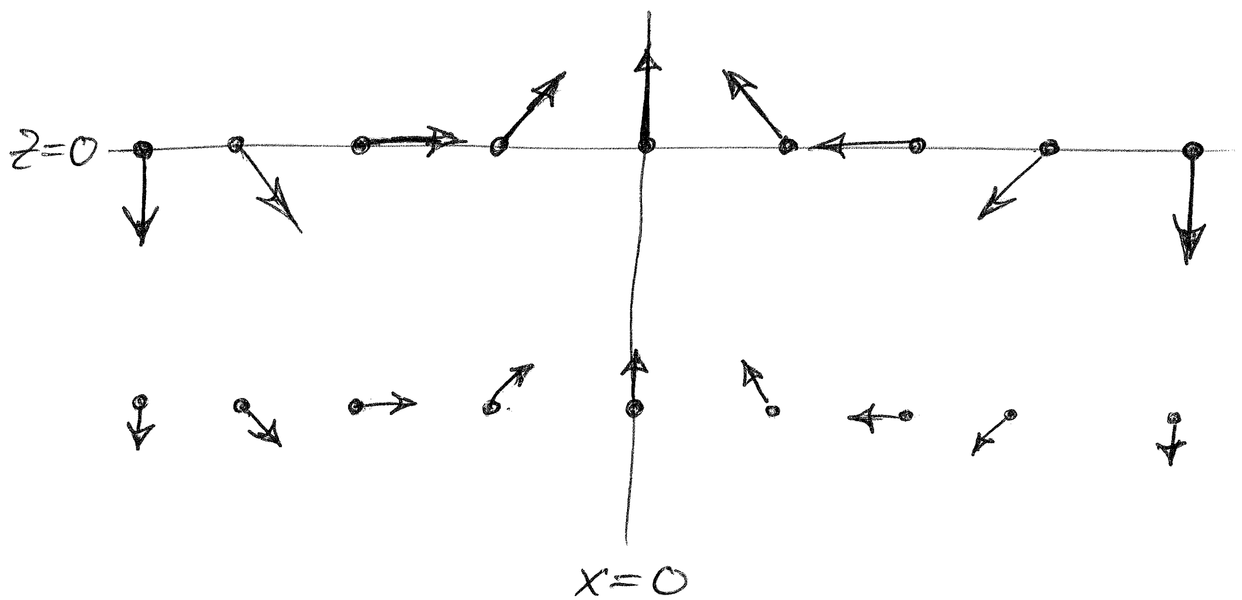
$$\Rightarrow \nabla^2 \phi = 0 \checkmark$$

(11.4)

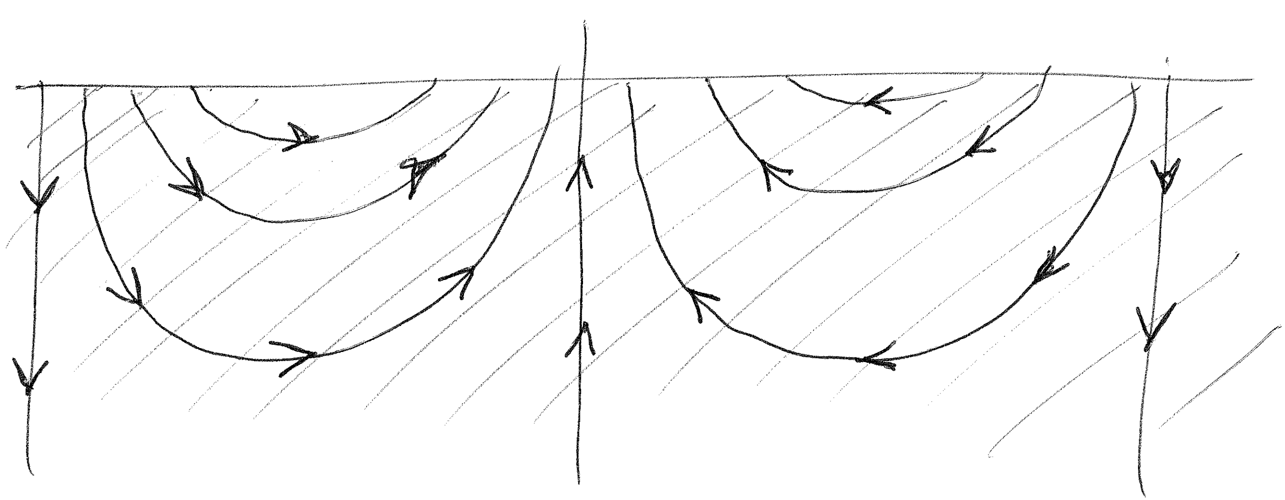
Sketch flow field:

$$V_x = \frac{\partial \phi}{\partial x} = -kA \sin kx \, e^{kz}$$

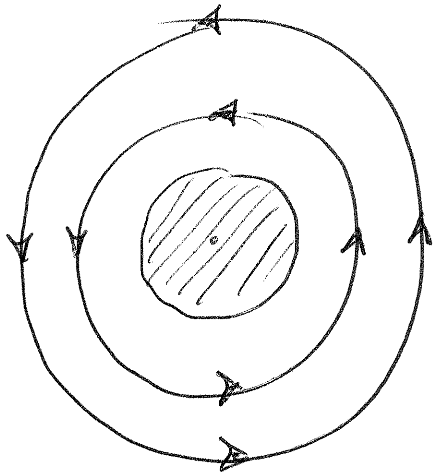
$$V_z = \frac{\partial \phi}{\partial z} = kA \cos kx \, e^{kz}$$



flow "field lines":



Aside : circular flow around cylinder



compare with $\vec{B}$ due to current flowing along cylinder (out of page)

$\vec{\nabla}\cdot\vec{B}=0\,,\quad \vec{\nabla}\times\vec{B}=0$

outside cylinder

Ampere's law $\Rightarrow$ $B \propto 1/r$

$\Rightarrow$ flow velocity decreases as $1/r$

$\Rightarrow$ individual fluid elements do not rotate (since $\vec{\nabla}\times\vec{v}=0$)