Assignment 1

Due date: Wednesday, January 29

Gyrating ring

A ring of radius $R$ rolls without slipping on a table while its axis is inclined with respect to the vertical by angle $\alpha$. As the ring gyrates, its center of mass stays fixed in space and its point of contact with the table moves around a circle with period $T$. Obtain an explicit expression for the ring’s angular velocity vector $\mathbf{\omega}$.

Rolling sphere

A sphere of radius $R$ rolls without slipping on a table. Interpret “rolling without slipping” in this case as the property that the point of the sphere making contact with the table is instantaneously at rest. Obtain a vector relationship between the sphere’s angular velocity $\mathbf{\omega}$, linear velocity $\mathbf{v}$, and the unit normal vector of the table, $\mathbf{n}$.

The number of “degrees of freedom” is usually defined as the number of continuous parameters required to uniquely specify the positions in a mechanical system. A better definition counts the number of independent velocity components of the system’s motion. According to the better definition, how many degrees of freedom does a rolling ball have?

Time dependent angular velocity

The body frame (of some body) is related to the space frame by the orthogonal matrix $U$. We learned that the time rate of change of $U$ satisfies the equation $\dot{U} = AU$, where

$$A = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

is an antisymmetric matrix whose nonzero elements correspond to the components of the angular velocity vector in the space frame. Consider a situation where $\mathbf{\omega}$ maintains a constant magnitude $\omega$ but changes its direction with time; in particular,

$$\omega_x = \omega \cos \Omega t \quad \omega_y = \omega \sin \Omega t \quad \omega_z = 0. $$

Suppose the space and body frames coincide at $t = 0$, so $U(0)$ is the identity matrix. Take a period of time $T = 2\pi/\Omega$, so $\omega$ completes one period. Over this period the average of the angular velocity vector is zero. Will $U(T)$ again be the identity matrix?
Write a simple computer program that implements a finite-difference integration of the equation $\dot{U} = AU$:

$$U(t + \Delta t) - U(t) = \Delta t A(t) U(t)$$

Check that your final $U(T)$ is nearly orthogonal; if not, you need to decrease $\Delta t$. Output $U(T)$ for two cases: $\omega = 0.1\Omega$ and $\omega = \Omega$. 