

Formulas

$$\begin{aligned}
U^T U &= U U^T = 1 \\
r &= Ur' \quad \dot{r} = \dot{U} U^T r = Ar \\
\dot{U} &= AU \\
A &= \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \\
\dot{\mathbf{r}} &= \boldsymbol{\omega} \times \mathbf{r} \\
\dot{\mathbf{a}} &= \overset{\circ}{\mathbf{a}} + \boldsymbol{\omega} \times \mathbf{a} \\
\mathbf{F}_{\text{fict}}/m &= -\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) - 2\boldsymbol{\omega} \times \overset{\circ}{\mathbf{r}} - \dot{\boldsymbol{\omega}} \times \mathbf{r} \\
T_{\text{rot}} &= \frac{1}{2} \sum_{\alpha, \beta} \omega_\alpha I_{\alpha\beta} \omega_\beta = \frac{1}{2} (I_1 \omega_1^2 + I_2 \omega_2^2 + I_3 \omega_3^2) = \frac{1}{2} \boldsymbol{\omega} \cdot \mathbf{I} \cdot \boldsymbol{\omega} \\
I_{\alpha\beta} &= \sum_i m_i (r_i^2 \delta_{\alpha\beta} - r_{i\alpha} r_{i\beta}) \quad \mathbf{I} = I_1 \hat{\mathbf{1}} \hat{\mathbf{1}} + I_2 \hat{\mathbf{2}} \hat{\mathbf{2}} + I_3 \hat{\mathbf{3}} \hat{\mathbf{3}} \\
L_\alpha &= \sum_\beta I_{\alpha\beta} \omega_\beta \quad \mathbf{L} = \mathbf{I} \cdot \boldsymbol{\omega} = I_1 \omega_1 \hat{\mathbf{1}} + I_2 \omega_2 \hat{\mathbf{2}} + I_3 \omega_3 \hat{\mathbf{3}} \\
\mathbf{N} &= \dot{\mathbf{L}} = \overset{\circ}{\mathbf{L}} + \boldsymbol{\omega} \times \mathbf{L} \\
I_1 \dot{\omega}_1 &= (I_2 - I_3) \omega_2 \omega_3 \\
I_2 \dot{\omega}_2 &= (I_3 - I_1) \omega_3 \omega_1 \\
I_3 \dot{\omega}_3 &= (I_1 - I_2) \omega_1 \omega_2 \\
\dot{\boldsymbol{\omega}} &= \Omega \hat{\mathbf{3}} \times \boldsymbol{\omega} \quad \Omega = (I_3/I - 1) \omega_3 \\
\dot{\hat{\mathbf{3}}} &= \omega_p \hat{\mathbf{L}} \times \hat{\mathbf{3}} \quad \omega_p = L/I \\
\omega_p \hat{\mathbf{L}} &= \boldsymbol{\omega} + \Omega \hat{\mathbf{3}} \\
\cos \theta &= \frac{I_3 \omega_3}{I \omega_p}
\end{aligned}$$