Diffraction through a slit of an obliquely incident wave

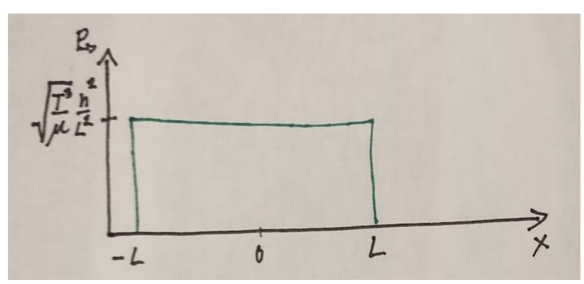
1. In this case, we have that the incident wave ($y < 0$) is $\psi = \Re\left(\tilde{A}e^{ik\sin(\theta)x + ik\cos(\theta)y}\right)$, for $|x| < a$, and the outgoing wave ($y > 0$) is given, as before, by $\psi = \Re\left(\int \hat{f}(k_x)e^{ik_x x + ik_y y}dk_x\right)$. At $y = 0$, requiring them to match gives $\tilde{A}e^{ik\sin(\theta)x} = \int \hat{f}(k_x)e^{ik_x x}dk_x$. We can determine $\hat{f}$ by
$$\hat{f}(k_x) = \int_{-a}^{a} \frac{\tilde{A}}{2\pi}e^{ik\sin(\theta)x - ik_x x}dx = \int_{-a}^{a} \frac{\tilde{A}}{2\pi}e^{i(k\sin(\theta) - k_x)x}dx$$
Let $q = k\sin(\theta) - k_x$, then this can be rewritten as $\int_{-a}^{a} \frac{\tilde{A}}{2\pi}e^{iqx}dx$, and we can recycle the result given in lecture finding
$$\hat{f}(k_x) = \frac{\tilde{A}}{\pi}a\frac{\sin(qa)}{qa} = \frac{\tilde{A}a}{\pi}\frac{\sin\left((k\,sin(\theta) - k_x)a\right)}{(k\,sin(\theta) - k_x)a},$$
Which is maximized when $k\sin(\theta) = k_x$. Since $k_x^2 + k_y^2 = k$, we can write $k_x = k\sin(\theta')$, and we finally wind that the maximum-magnitude amplitude occurs at
$$\theta' = \theta.$$

Diffraction of waves around an obstacle

1. The function $f(x)$ is given by
$$f(x) = \begin{cases} 0 & |x| < a \\ \tilde{A} & |x| > a \end{cases}.$$
2. Without the block, the function $f_0(x) = \tilde{A}$. The difference is then
$$\Delta f(x) = f(x) - f_0(x) \begin{cases} -\tilde{A} & |x| < a \\ 0 & |x| > a \end{cases}.$$
3. The difference function is, up to a minus sign, the same as the function defined by the slit, so the Fourier transform is simply
$$\Delta\hat{f}(k_x) = -\frac{\tilde{A}}{\pi}a\frac{\sin(k_x a)}{k_x a}$$
4. The unobstructed wave function $\tilde{\Psi}_0 = \tilde{A}e^{iky}$, and the difference function is $\int -\frac{\tilde{A}}{\pi}a\frac{\sin(k_x a)}{k_x a}e^{ik_x x + k_y y}\,dk_x$, so the diffracted wave function
$$\tilde{\Psi} = \tilde{A}e^{iky} - \int \frac{\tilde{A}}{\pi}a\frac{\sin(k_x a)}{k_x a}e^{ik_x x + k_y y}\,dk_x.$$

Power in a stretched string

1. The power is given by $P_\rightarrow = -T\frac{\partial y}{\partial x}\frac{\partial y}{\partial t}$. For $y = f(x - vt)$, we have $\frac{\partial y}{\partial x} = f'(x - vt)$ and $\frac{\partial y}{\partial t} = -vf'(x - vt)$, so $P_\rightarrow = vT[f'(x - vt)]^2$. At $t = 0$ we then have $P_\rightarrow = vT[f'(x)]^2$. For a triangle pulse, $f(x) = h + \begin{cases} \frac{h}{L}x & -L < x < 0 \\ -\frac{h}{L}x & 0 \leq x > L \end{cases}$, which



implies $P_{\rightarrow} = vT\frac{h^2}{L^2} = \sqrt{\frac{T^3}{\mu}}\frac{h^2}{L^2}$ for $|x| < L$ at $t = 0$, and 0 otherwise.

2. The energy passing through the point is the integral of the power over time. The total time it takes for the pulse to travel through the point is $\frac{2L}{v} = 2L\sqrt{\frac{\mu}{T}}$. The energy is then

$$E = \int_0^{2L/v} vT\frac{h^2}{L^2}dt = 2T\frac{h^2}{L}$$

Generalizing the intensity formula of a sound wave

We guess that the generalization is $\boldsymbol{I} = -B(\nabla \cdot \boldsymbol{s})\frac{\partial}{\partial t}\boldsymbol{s}$. This satisfies the requirements: the intensity is a vector, since $-B(\nabla \cdot \boldsymbol{s})$ is a scalar and $\frac{\partial}{\partial t}\boldsymbol{s}$ is a vector, it is in terms of a vector field $\boldsymbol{s}$, the equation is valid for any coordinate system, and since in cartesian units $\nabla \cdot \boldsymbol{s} = \frac{\partial}{\partial x}(\boldsymbol{s} \cdot \hat{\boldsymbol{x}}) + \frac{\partial}{\partial y}(\boldsymbol{s} \cdot \hat{\boldsymbol{y}}) + \frac{\partial}{\partial z}(\boldsymbol{s} \cdot \hat{\boldsymbol{z}})$, for the case $\boldsymbol{s} = s\,\hat{\boldsymbol{x}}$, the becomes $\frac{\partial s}{\partial x}$, and we get $I\,\hat{\boldsymbol{x}} = -B\frac{\partial s}{\partial x}\frac{\partial s}{\partial t}\,\hat{\boldsymbol{x}}$, or $I = -B\frac{\partial s}{\partial x}\frac{\partial s}{\partial t}$.