Homework 9

Due date: Wednesday, April 17

Diffraction through a slit of an obliquely incident wave

Repeat the slit diffraction calculation in lecture (absorbing screen), but where the wave vector of the incident wave has angle $\theta \neq 0$ with respect to the screen-normal. At what angle θ' will the diffracted wave have its maximum-magnitude amplitude?

Diffraction of waves around an obstacle

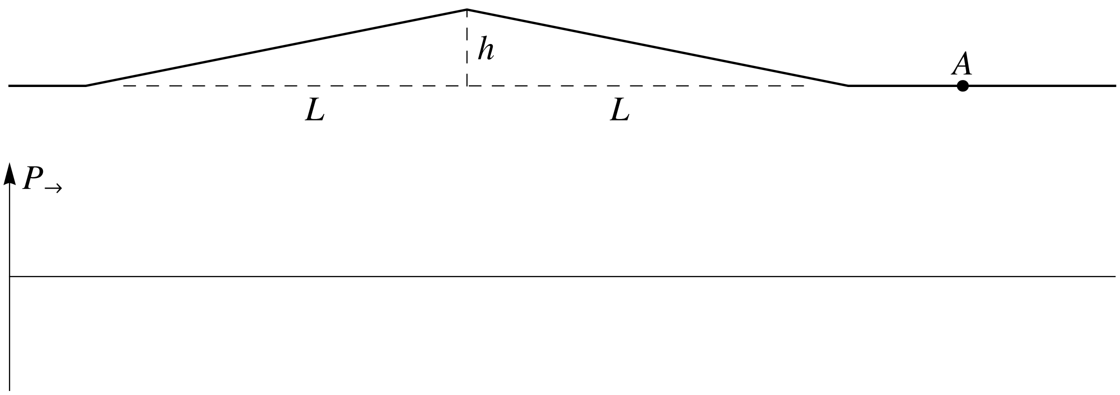
Consider the complete opposite of the slit-diffraction geometry. Instead of an aperture of length $2a$ in an infinite absorbing screen, there is a length $2a$ piece of absorbing screen blocking the wave, and nothing else. As in the slit-diffraction geometry, a wave with wave vector $k\hat{\mathbf{y}}$ is incident in the region $y < 0$. What happens?

Work this out exactly as in lecture for the slit. However, instead of calculating the waves in the region $y > 0$, now calculate the **change in the waves** caused by the "block". That is, if $\tilde{\Psi}$ is the total wave, and $\tilde{\Psi}_0$ is the wave in the absence of the block, your task is to calculate $\Delta\tilde{\Psi} = \tilde{\Psi} - \tilde{\Psi}_0$.

1. What is the function $f(x)$ defined by the incident wave on the $y = 0$ "plane" when the block is taken into account?
2. What function $\Delta f(x)$ corresponds to the difference $\Delta\tilde{\Psi} = \tilde{\Psi} - \tilde{\Psi}_0$, again on the $y = 0$ plane?
3. What is its Fourier transform, $\Delta\hat{f}(k_x)$?
4. $\Delta\hat{f}(k_x)$ is the **complex amplitude** of the difference-wave $\Delta\tilde{\Psi}$ in the region $y > 0$ with wave vector $k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}}$ where $k_y = \sqrt{k^2 - k_x^2}$. Describe the combined waves $\tilde{\Psi}$ in $y > 0$ given that $\tilde{\Psi} = \Delta\tilde{\Psi} + \tilde{\Psi}_0$.

Power in a stretched string

A triangular pulse is propagating to the right on a stretched string of tension T and mass per unit length μ. The pulse shape at time $t = 0$ is shown below:



1. Plot the instantaneous power at $t = 0$ in the pulse, $P_{\rightarrow}$, directly under the pulse.
2. Determine the total energy that has passed through point A after the entire pulse has moved through this point.

Generalizing the intensity formula of a sound wave

In lecture we worked out that

$$I = -B \frac{\partial s}{\partial x} \frac{\partial s}{\partial t} \tag{1}$$

is the intensity of a one-dimensional sound wave in a medium with bulk modulus B. That is, I equals the energy per unit time and area passing through a planar surface perpendicular to the x-axis, and in the positive direction (when $I < 0$ it means the energy is moving in the negative direction). For a one-dimensional sound wave $s(x, t)$ is the x-displacement of the particles in the medium.

Generalize formula (1) when particle displacements are not necessarily along the x-axis (say near a source, where the waves are moving radially away from the source). Here are some things to keep in mind when solving this problem:

- **Do not re-derive the intensity formula from scratch!**
- Your formula should be expressed in terms of a **vector field** of particle displacements, $\mathbf{s}(\mathbf{r}, t)$. In other words, it should be mathematically sensible for a general vector field.
- **Your formula should itself be a vector**. Just like the intensity-vector of an electromagnetic wave[1] points in the direction of energy flow, your intensity-vector for sound should also point in the direction of energy flow.
- Guess a vector-calculus generalization of (1) that could be used in any coordinate system. That is, there should be nothing special about x.
- Check that your generalization reduces to (1) for the special case

$$\mathbf{s}(\mathbf{r}, t) = s(x, t)\, \hat{\mathbf{x}} \, .$$

[1] Usually written $\mathbf{S}$ and called the "Poynting" vector.