Intrinsic localized modes (ILMs) in the coupled pendulum system

1. The frequency range was found from the equation which was derived assuming small angles, which is no longer valid in this case.
2. Treating all other oscillators as small amplitude pendulums, the frequency range is still valid for these. Since the driving pendulum has a smaller frequency than any of these frequencies, the driving force will not be in resonance with any of the modes, and therefore their amplitudes will remain small.

Volume dependence of the kinetic energy of a 3D gas of point particles

1. As stated in the homework, we follow the same steps as for the 2D case. The change in energy for a single particle with velocity v_x is $-2mv_xV_x$, where V_x is the speed of the wall. The fraction of particles with velocity between v_x and $v_x + dv_x$ is $f(v_x)dv_x$. The fraction of particles within range for colliding in a time t is $\frac{v_x \Delta t}{L}$ (under the assumption $V_x \ll v_x$). The total number of particles that can collide is then with velocity between v_x and $v_x + dv_x$ that undergoes a collision is then $N\frac{v_x\Delta t}{L}\ f(v_x)dv_x$, and therefore the change in energy for these particles is $dE = -2mN\frac{V_x\Delta t}{L}v_x^2 f(v_x)dv_x = -2mN\frac{\Delta L}{L}\ v_x^2 f(v_x)dv_x$. The total change in energy is then $\Delta E = -2mN\frac{\Delta L}{L}\int_0^\infty v_x^2 f(v_x)dv_x = -mN\frac{\Delta L}{L}\int_{-\infty}^\infty v_x^2 f(v_x)dv_x = -mN\frac{\Delta L}{L}\langle v_x^2\rangle$. Now comes the difference: isotropy in 3D implies $\langle v_x^2\rangle = \langle v_y^2\rangle = \langle v_z^2\rangle$, and therefore the total energy $E = \frac{1}{2}Nm\left(\langle v_x^2\rangle + \langle v_y^2\rangle + \langle v_z^2\rangle\right) = \frac{3}{2}Nm\langle v_x^2\rangle$ (as opposed to $Nm\langle v_x^2\rangle$ as in the 2D case). This means $\frac{\Delta E}{E} = -\frac{2}{3}\frac{\Delta L}{L}$. Integrating, we find $\ln\left(\frac{E}{E_0}\right) = \frac{2}{3}\ln\left(\frac{L_0}{L}\right) = \ln\left(\left(\frac{L_0}{L}\right)^{\frac{2}{3}}\right)$, or

$$E(L) = \left(\frac{L}{L_0}\right)^{-\frac{2}{3}} E(L_0).$$

Uncertain tuning

1. We know that $\Delta k\Delta x = const.$, where the specific constant on the right will depend on the particular shape of the pulse in time. For example, for a gaussian or a uniform pulse, we found $const. = 2$, and we can use that in this estimate (it seems reasonable to assume the oboe will play at a constant intensity over the given period of time). The specific number isn't very important anyway, since we are looking for an order of magnitude estimate. Because sound has a linear dispersion in air, we know $\omega = c_s k$, where c_s is the speed of sound in air, which implies $\Delta\omega = c_s\Delta k$. Since $c_s = \left|\frac{dx}{dt}\right|$, we have $\Delta t = \frac{\Delta x}{c_s}$, and therefore $\Delta\omega\Delta t = const$. Since $f = \frac{\omega}{2\pi}$, $\Delta f = \frac{\Delta\omega}{2\pi}$, and $\Delta f\Delta t \sim \frac{1}{\pi} \to \Delta t \sim \frac{1}{\pi\Delta f}$. For $\frac{\Delta f}{f_A} \approx 10^{-3}, \Delta f \approx 10^{-3} f_A = 0.44\ s^{-1}$ (keeping significant figures for now to avoid introducing too much error). This gives

$$\Delta t \sim \frac{1}{0.44\pi} s \approx 0.72\ s \approx 1\ s.$$

Mean free path of spherical particles

1. The particles themselves are spheres with radius r, but the two particles will collide if their cross-sectional areas overlap, which will happen if the two particles' centers are within a distance $2r$ of each other. Therefore, if the moving particle is considered a point, the "targets" can be considered disks with area $\pi(2r)^2 = 4\pi r^2$.
   The probability of a particle being in a given length l is equal to the ratio $\frac{l}{L}$, where L is the total length of the volume containing the gas. The probability of a particle being in a collision path with the moving particle is $\frac{4\pi r^2}{A}$, where A is the area of the volume. The number of particles on a collision course is $N\frac{4\pi r^2}{A}\frac{l}{L} = \frac{N}{V}4\pi r^2 l = 4\pi r^2 nl$. We get even odds of a collision when this is equal to 1, so $4\pi r^2 nl = 1$, or
   $$l = \frac{1}{4\pi r^2 n}.$$
2. The number density of the particles can be obtained from the ideal gas law, which states $PV = Nk_BT$, which rearranged gives $\frac{N}{V} = n = \frac{P}{k_BT} \approx 10^{25}/\mathrm{m}^3$. The radius r is likely on the order of the atomic radius, which is on the scale of $10^{-10}\mathrm{m}$. With $r \approx 10^{-10}\mathrm{m}$, we have
   $$l \approx \frac{1}{4\pi\ 10^{-20} \times 10^{25}}\ \mathrm{m} \approx 0.08 \times 10^{-5}\ \mathrm{m} \approx 1\ \mu\mathrm{m}.$$