Homework 7

Due date: Wednesday, March 20

Intrinsic localized modes (ILMs) in the coupled pendulum system

As promised, this exercise will help you understand the phenomenon of *intrinsic localized modes*. If the system of coupled pendulums behaved like the other systems we've seen, like water-surface waves, then a large amplitude initial condition, that starts out localized, eventually spreads out because of wave propagation (and is then no longer localized). But as you can see, this does *not* happen in the animation on the course website!

1. In the last homework you found the range of possible frequencies $\omega(k)$ for wave propagation in the coupled pendulum system. What did that analysis leave out that explains why a single *large amplitude* pendulum can have a frequency ω *smaller* than the $\omega_{\min}$ of the dispersion relation $\omega(k)$?
2. In a few sentences explain the ILM phenomenon by treating the central, large amplitude pendulum, and all the other small amplitude pendulums, separately. Think of the former as "driving" the latter.

Volume dependence of the kinetic energy of a 3D gas of point particles

Repeat the derivation in lecture of the volume dependence of the kinetic energy of a gas when the "cylinder" enclosing the gas is a true 3D cylinder with cross-sectional area A and length L (in lecture the enclosure was 2D, with a 1D cross-section). The 2D and 3D derivations deviate only when it comes to relating the change in energy due to the moving wall (changing L) to the total kinetic energy of the gas by invoking the isotropy principle. As in the 2D case, your answer should have the form

$$E(L) = E(L_0) f(L/L_0) \ ,$$

where $E(L_0)$ is the energy when the cylinder length is L_0 and $f(x)$ is a simple function you can determine explicitly (with the property $f(1) = 1$).

Uncertain tuning

The instruments of an orchestra are usually tuned to agree with the frequency $f_A = \omega_A/(2\pi) = 440$ Hz, the A above middle C. As a student of Physics 2218 you know, however, that the "purity" of a musical tone is compromised when it has a finite duration, *i.e.* it will actually comprise a superposition of tones with a range of frequencies, Δf. Estimate (order-of-magnitude) how long the oboe must play an A in order to define its frequency to within 0.1% ($\Delta f/f_A \approx 10^{-3}$). Sound in air has a linear dispersion relation.

Mean free path of spherical gas particles

1. The mean free path ℓ of particles in a gas is the typical distance a gas particle moves before it collides with another gas particle (scrambling their velocities in the process). While it is possible to define a mean or average distance rigorously in a mathematical sense, in this problem we care only how this distance depends on two parameters: the number density n of the particles (number per unit volume) and their size. To keep things simple, you'll consider spherical gas particles whose size is set by their radius r. Your goal is to arrive at a formula $\ell = f(n, r)$, where the function f has units of length, *and* you can make the case that $f(n, r)$ is a rough estimate of the mathematical mean.

   Here are some suggestions to help you get started:

   - Consider instead the simpler but related problem of one moving particle and all other particles at rest at random positions (but with the specified density).
   - Think of the stationary particles as presenting "targets" to the moving particle. What is the shape and size of these targets?
   - What ℓ gives "even odds" that a target is hit?

2. Make an order-of-magnitude estimate of the mean free path of a molecule in air. Instead of just quoting values of n and r you found in Wikipedia, also come up with a strategy for retrieving these numbers (at the order-of-magnitude level) from memory.