Homework 4

Due date: Wednesday, February 21

Co-ax boundary conditions

1. Based on what you learned in the previous homework, write down the most general expression for the surface charge density σ on the inner conductor of an infinite co-ax (a function of z and t).
2. What surface current density $j\hat{\mathbf{z}}$ (also on the inner conductor) corresponds to your expression for σ ? Tip: use conservation of charge.
3. What boundary condition, on σ or j, applies to an "open" end at $z = 0$, that is, when the two conductors remain disconnected? How does this restrict the form of the general solution?
4. What boundary condition, on σ or j, applies to a "shorted" end at $z = 0$, when the two conductors are joined? How does this restrict the form of the general solution?
5. The oscilloscope measures the voltage between the conductors, so indirectly σ. If a positive voltage pulse is launched toward an open end, what will be the sign of the reflected pulse? Sketch the corresponding oscilloscope trace you saw in lecture.
6. If a positive voltage pulse is launched toward a shorted end, what will be the sign of the reflected pulse? Sketch the corresponding oscilloscope trace you saw in lecture.

Hanging chain normal modes

In homework 1 you derived the equation for transverse motion of a hanging chain:

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial z}\left(gz\frac{\partial y}{\partial z}\right) .$$

1. Use the normal mode form for y,

$$y(z,t) = \cos(\omega t) f_\omega(z) ,$$

   and obtain the differential equation (in just the variable z) satisfied by f_ω.

2. Show that your differential equation is solved by

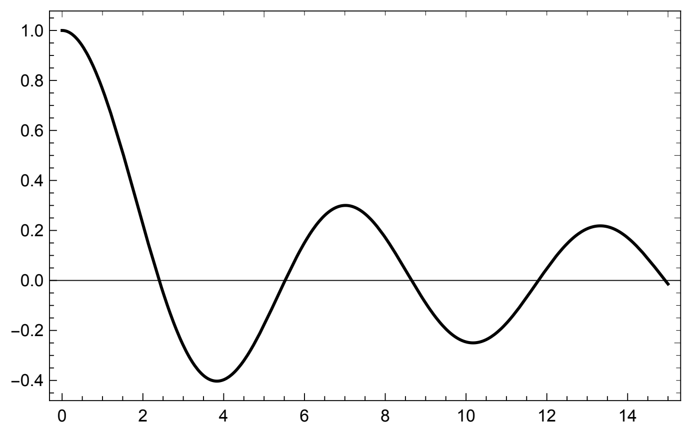
$$f_\omega(z) = J_0(c_\omega \sqrt{z}) \ ,$$

where J_0 is the "Bessel function of order zero" [1] and c_ω is a particular constant. The only property of J_0 you need to answer the question is its defining differential equation:

$$x \frac{d^2 J_0}{dx^2} + \frac{dJ_0}{dx} + x J_0 = 0 \ .$$

What is the value of c_ω ?

3. Here is a plot of $J_0(x)$ over the range $0 < x < 15$:



Unlike the sine and cosine functions, the zeros of J_0 are not evenly spaced. Use the following numerical values for the first few zeros,

2.40483 5.52008 8.65373 11.7915 14.9309

to obtain the *periods* of the first three normal modes for a chain of length L. Like the simple pendulum, by unit analysis the normal mode periods must be numerical multiples of the time scale $\sqrt{L/g}$.

[1] This function makes an appearance later in the course when we study the diffraction of light through a circular aperture

Circuit model for myelinated nerve fibers

Derive a circuit model for myelinated nerve fibers (the axon of a neuron) much like we derived a circuit model for the co-ax cable. Refer to the drawing in the lecture notes for the geometry and relevant sizes. Unlike the co-ax, the individual circuit elements of a myelinated nerve fiber are explicitly defined by the segments of myelin that surround the fiber. Think of the myelin as very thick insulation, so the only locations for capacitative charge storage are the much thinner, small bands of exposed cell membrane called the *Nodes of Ranvier*.

1. What is the capacitance C of one of the *nodes*? Since the cell membrane thickness is so much smaller than all the other scales you can use the parallel plate model. Use $\kappa = 7$ for the dielectric constant of the cell membrane.
2. Model the "insulated wire" between consecutive *nodes* as a resistor . What is the resistance R given that the resistivity ρ of the axoplasmic medium is about $1\,\Omega$ m.
3. Using the notation $i(n,t)$ as the current in the n^{th} wire (as we did for the co-ax), derive a partial differential equation for i. The time scale of your equation should come out to be RC. What is the value of RC in seconds? Relate your answer to human unconscious reaction time. Consider pianists that play *Flight of the Bumblebee* at 20 notes per second[2] !
4. Does your equation for myelinated nerve conduction have time-reversal symmetry?

[2] And that probably involves signals in the brain as well.