Homework 2

Due date: Wednesday, February 7

Coupled pendulums

Later in the course we will study a wave phenomenon that was discovered by retired Cornell physics professor Albert Sievers in the 1980s. In this exercise you derive the equations of motion for a table-top realization of the effect. The effect, called *intrinsic localized modes* (ILMs), has been observed in diverse physical systems.

Identical pendulums swing about a common axis, not unlike the rods of the torsional wave apparatus you've seen in lecture. And just like that system we use angles $\theta_n(t)$ to describe the pendulums at any time t. Every pendulum has the same moment of inertia I about the rotation axis, and there is a spacing Δx between the pendulums. The pendulums are coupled exactly as the rods of the torsional wave apparatus, by torsional springs that have a linear torque-angle relationship with slope κ. The only new feature is the fact that gravity plays a role. Instead of two perfectly balanced rod-halves, each pendulum is a single rod that prefers to hang down.

1. Every pendulum has total mass m and the center of mass is located a distance r from the axis of rotation. Determine the torque due to gravity on pendulum n. Do *not* assume θ_n is small.
2. Derive the equations of motion for the system of coupled pendulums. Do not make any approximations other than the linearity of the torque-couplings of adjacent pendulums. Your equations will involve the following parameters: I, κ, m, r and g (acceleration of gravity).
3. Approximate your equations for the case when all the θ_n are small (all the pendulums are hanging nearly straight down). The ILM effect goes away in this approximation, and we will try to understand (later) why this is the case.
4. Go even further, and apply the approximation where the discrete variables $\theta_n(t)$ are replaced by continuous ones, $\theta(x, t)$, exactly as in lecture. This introduces another parameter, Δx, and the result will be a partial differential equation, but *not* the wave equation.

Co-tape™

Two former 2218 students dropped out of Cornell shortly after (successfully) completing the course to start *Tapetronix*, a company whose main product, *Co-tape*, is a cheap alternative to the standard co-axial cable. *Co-tape* is nothing more than a thin ribbon of aluminum foil with an adhesive backing in a convenient spool dispenser. To connect two electrical devices, assumed already mounted on a good conducting frame (car, etc.), you simply "tape" a connection between them, applying the (insulating) adhesive side of the *Co-tape* to the metal frame.

Model *Co-tape* as a conducting strip of width w and negligible thickness. The adhesive is a good insulator with dielectric constant κ and maintains a distance d from the metal surface to which it is attached. Because the charge and current fluctuations in the tape are bounded in extent by its width w, you may assume the same restriction applies to the charge and current fluctuations in the metal underneath the tape. Finally, because the wavelength of wave that *Co-tape* is used for is long compared to w, the charge fluctuations are modulated (and current flows) only along the length of the tape (as in the co-ax model).

1. Calculate the capacitance per unit length of tape.
2. Calculate the inductance per unit length of tape. As in the capacitance calculation, you can assume the (electric/magnetic) field is uniform between the two closely spaced metal sheets (neglect "edge effects").
3. What is the velocity of waves in *Co-tape* ?