Assignment 11

This is the final assignment and should be submitted by Tuesday, November 28, to the Physics Office.

Non-adiabatic loss from a magnetic trap

Using the results of the last problem of assignment 10 and Fermi's Golden Rule, complete the calculation begun in lecture, of the differential rate

$$\frac{d\Gamma}{d\Omega} = C f(\theta, \phi)\, e^{-2\omega_p/\omega_0}$$

at which the trapped spin-1/2 particle exits the trap along polar angles (θ, ϕ) with respect to the direction of the magnetic field at the center of the trap. Express C only in terms of fundamental constants, the particle mass m and magnetic moment μ, and the oscillator and precession frequencies (ω_0 and ω_p) instead of the trap parameters (isotropic case).

Interpret the angular function $f(\theta, \phi)$ in terms of a conservation law.

Gauge transformation of the magnetic field in Yang-Mills theory

Apply the Yang-Mills generalization of gauge transformations,

$$A'_x = U A_x U^\dagger - i\, U(\partial_x U^\dagger)$$
$$A'_y = U A_y U^\dagger - i\, U(\partial_y U^\dagger)$$

to the Yang-Mills generalization of the magnetic field

$$B_z = \partial_x A_y - \partial_y A_x + i[A_x, A_y].$$

Treat A_x, A_y, B_z and U as non-commuting matrices, as in the analysis of the magnetic trap. Using only the group property $UU^\dagger = \mathbb{1}$ (and its derivatives) show that

$$B'_z = U B_z U^\dagger.$$