Assignment 1

Due date: Wednesday, February 28

Naive difference-of-projections flow

Argue, with the help of a simple example, why the dynamical system

$$\dot{x} = P_A(x) - P_B(x)$$

will fail to *find* a point $x^* \in A \cap B$, even though such a point is a fixed point of the flow.

Square and Line

Hypercubes and hyperplanes find many applications as constraint sets. Consider the toy example in two dimensions, where $A = \{-1, 1\} \times \{-1, 1\}$ and $B = \{(s, s) : s \in \mathbb{R}\}$. Sketch the RRR flow for this pair of constraint sets and identify the set of fixed points and their associated solutions. Do all trajectories lead to fixed points or are there cycles?

Histogram projection

Suppose you have a grayscale image and wish to change the distribution of pixel values. How can this be done to minimize the change, in the sense of a constraint projection?

Say the original image — flattened out — has pixel values $\{x_1, \ldots, x_N\}$ and when modified has values $\{p_1, \ldots, p_N\}$. When sorted, it is easy to see what the second list would have to be for any target distribution. For example, if we want a perfectly uniform distribution of gray scales we would use $\{p_1, \ldots, p_N\} = \{1, \ldots, N\}$ (or a scaled version of this).

Here's the algorithm for projecting a flattened image $\{x_1, \ldots, x_N\}$ to a target list of sorted pixel values $\{p_1, \ldots, p_N\}$. First find the ordering r of the $x's$. For example, $r(x_{14}) = 42$ means that x_{14} would be in position 42 when the x's are sorted. The projection is then simply

$$\{x_1, \ldots, x_N\} \to \{p_{r(x_1)}, \ldots, p_{r(x_N)}\}.$$

Prove that this is a constraint projection, i.e. that this minimizes the Euclidean distance

$$\sum_{i=1}^{N} \left| x_i - p_{r(x_i)} \right|^2 .$$

Write a program that uniformizes the pixel distribution of gray scale images and apply it to the Escher image `DrawingHands.jpg`.