# Assignment 3

Due date: **Friday**, February 17

*Rigid body simulation*

A rigid body has principal moments

$$I_1 = 1 \quad I_2 = 2 \quad I_3 = 3$$

in appropriate units. The body's angular velocity in the body basis

$$\boldsymbol{\omega}(0) = \omega_1(0)\hat{\mathbf{1}}(0) + \omega_2(0)\hat{\mathbf{2}}(0) + \omega_3(0)\hat{\mathbf{3}}(0)$$

has value

$$\omega_1(0) = 1 \quad \omega_2(0) = 1 \quad \omega_3(0) = 0.9$$

at time $t = 0$ in appropriate units. Write a loop in your favorite computer language to time-evolve the body-frame angular velocity components by Euler's equations for one orbit. To decide when you have completed one orbit compute the distance (in angular-velocity space)

$$\|\boldsymbol{\omega}(t) - \boldsymbol{\omega}(0)\|$$

and terminate if this is less than 0.01 and $t > 1$. Use $dt = 0.001$ as the time increment in the loop. You should find a period close to $T = 7.2$.

Add a few more lines to your loop that time-evolve the body basis vectors using

$$\dot{\hat{\mathbf{1}}} = \boldsymbol{\omega} \times \hat{\mathbf{1}} \qquad \dot{\hat{\mathbf{2}}} = \boldsymbol{\omega} \times \hat{\mathbf{2}} \qquad \dot{\hat{\mathbf{3}}} = \boldsymbol{\omega} \times \hat{\mathbf{3}}.$$

Print out the body basis vectors after one period of the motion (at time $t = T$). Check that you still have an approximately orthonormal basis. Finally, find the rotation angle $\theta$ of the body after one period using the trace property of the rotation matrix $U$ (that relates initial and final basis vectors):

$$\mathrm{Tr}\, U = 1 + 2\cos\theta.$$

*Frictionless ladder*

Write down the equation of motion for the single degree of freedom, the angle $\theta$, of the frictionless ladder leaning against a moving wall, as discussed in lecture. Show that the equation has a solution where $\theta$ is constant when the motion of the wall has a special form.